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Mathematics Education: Implications from Educational Psychology

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Abstract

Mathematics education is a major part of the realm of education today. However, the utilization of educational psychology in this area is often neglected. This paper will discuss some of the relevant realms of educational psychology that are often neglected in mathematics education and also provide some insights into the improvement of mathematics education.

Introduction

"Reading, writing, and arithmetic, "often referred to as the 3R's, have been around the educational world for many years. Upon closer examination, one notes that arithmetic (or "maths" as it is referred to in England) is third in this triad. There are undoubtedly sound reasons for this, as many students detest math and calculations. Many teachers abhor its teaching, and many students suffer exasperation, frustration, and pure angst as they attempt to understand and comprehend "arithmetic" or math.

The reasons for the almost universal hatred of math are many and will be explored here. While some students are able to use their fingers and count to ten, additional math mastery and performance seem to stop there. This paper will first review some global issues related to the learning of numerical operations, then discuss some intervention strategies. The paper does not attempt to solve all the problems but attempts to provide some cognitive structures in terms of grasping the problems involved in teaching and learning arithmetic.

1) Basic Concepts and Vocabulary: In the early years of a child's life, there is a rote attempt on the part of parents to explain the concept of simple numbers from one to ten- this being because the child has ten fingers- 5 on each hand and thus it makes perfect sense to prompt the child to count to ten, much as the repeat the alphabet in a sing song like manner. Along with counting to ten comes some minimal vocabulary. For example, the words "more "and "less" are often used in everyday conversation but not always explained by the parents. Often, the child will see a concept demonstrated visually by the parent, e.g., more cookies, fewer cookies, but not all parents engage in this form of direct instruction. Hence students arrive at kindergarten or first grade with only a surface comprehension of what has been termed "basic concepts." There are, in fact, a number of rating scales and tests, such as the Boehm Test of Basic Concepts, which is designed to ascertain a student's grasp of basic numerical concepts. There are a number of other early childhood assessments that, while endeavoring to assess expressive and receptive language, indirectly measure many of these basic concepts of mathematical terms (such as more, greater than lesser than), and this issue of basic concepts and vocabulary continues on into grade school and elementary school as some children have difficulty with vocabulary words such as subtract, add, multiply, divide. If children do not have a firm comprehension of these basic words (such as dividend and subtrahend), then the students of below-average intelligence may feel lost and alienated from their peers and their subject matter.

2) Concrete Learners: Some children have a very basic concrete learning style. Teachers or parents have to place ten pennies or ten pieces of candy in front of the child to demonstrate counting and the process of addition and or subtraction (or, as many kids refer to it today as "take away." For some children seeing the cookies being taken away from them (especially when they are hungry) rings true and hits home. The problem later becomes when students are confronted with larger numbers and the teacher can no longer put 100 pennies in front of the child to "take away" ten or so to demonstrate subtraction.

3) English Language Learners have substantial difficulty with math inasmuch as they are still in the process of learning the English language. They may be speaking Spanish in the home, listening to parents speak in Spanish, and listening to Spanish music or television. Thus, they are continually attempting to grasp what a rapidly talking math teacher is saying or instructing them to do.

4) In addition to inherent complexity with mathematical concepts, culturally contingent sources of difficulty may exist due to cultural values and characteristics of educational systems. Lortie-Forgues et al. (2015) discuss several potential sources of difficulty relating to cultural values in their review article. The two main categories of problems studied include factors related to instruction and factors related to the learners' prior knowledge. How a culture distinguishes required and optional skills may, in part, influence students' mathematical performance.

5) The Dual Process Theory is another area of psychological research that focuses on the relationship between the innate and diagnostic modes of thinking and behavior. Leron & Hazzan (2006) discuss the details of this theory and explain its effect on student learning and problem-solving. In summary, this theory states that our cognition and behavior function analogously in two entirely different ways, called System 1 (S1) and System 2 (S2), approximately equivalent to our common sense philosophies of intuitive and analytical thinking. Different parts of the brain activate S1 and S2. S1 is depicted as being rapid, spontaneous, reflexive, effortless, and automatic. In contrast, S2 is deliberate, mindful, labor-intensive, and relatively adaptable.

6) When implemented simultaneously, MCII and the Dual Process Theory may offer educators a valuable intervention technique when teaching certain mathematical concepts, such as fractions, decimals, and percentages, as there are intricacies integral to fraction and decimal arithmetic. According to the findings of Lortie-Forgues et al. (2015), interventions directed at helping children overcome difficulties inherent to decimal and fraction arithmetic can produce significant gains in performance and understanding.

7) Models, exemplars, and examples- Demonstrations are imperative for some learners in the realm of math. If a teacher gives a clear, step-by-step demonstration of what the child needs to do, then for many students, much can be expected of them once they grasp what the model is trying to indicate. Often a teacher will bring a student to the blackboard or overhead transparency and have them "walk through" the process of multiplication and or division so that other students can hear what the thinking of the student is like. Often peer instruction is the best instruction. Quite often, the graduate assistant in college statistics classes is able to provide more clear instruction than the Ph.D., who is invariably talking over the heads of the students.

8) Formative Evaluation or Assessment: One critical aspect of teaching math is the need for an initial formative evaluation or assessment of the present level of performance of student math skills. Math teachers need to assess at the beginning of each grade- how much students may have mastered from the previous grade, and how much they may have forgotten over the summer months. Recently, this idea of "summer decline" has become increasingly important and discussed and needs to be increasingly investigated. Math skills tend to "decay" or vanish if not utilized, and while a student may have been perhaps 70 % proficient at the end of one grade, their skills may have deteriorated over the summer months and need review. Not all teachers have the time or the inclination to do this.

9) Review of standardized test scores: Teachers do have access on occasion to student data in the child's file. It is imperative for teachers to acquaint themselves with the student's progress and current level of functioning. While these grade or age equivalents are basically global representatives of student functioning, extreme problems or deficiencies can be ascertained and addressed. There is a concept that has been bandied about for several years called "Response to Intervention." The basic premise surrounding this is that when a teacher ascertains a student who is struggling, the teacher may put that child into a small group for remediation or perhaps, if there is an Educational Aid or assistant available, to ask the aid to work with the student on their deficiencies or weak areas. Should the child not seem to respond to the Educational Aid or assistant, then the teacher may want to work with the child one on one, before school, after school, during the child's art or music, or Physical education class in an attempt to remediate- perhaps assigning additional homework and contacting the parent about the student's weak areas. These weaknesses may be due to poor instruction or the teacher's absence the previous year.

10) The Understanding of Numerical Concepts: For some students, there is a lack of understanding of numerical concepts at the elementary level. For example, even the idea of an “even” and an “odd” number present challenges to some students. This continues on, and at some point, they are confronted with “prime numbers” and “square roots.” For some students, the teacher is speaking in a different language- unless things are clearly, slowly explained.

11) Children need to develop an understanding of equivalence. This process should begin with concrete experiences, like manipulating objects to form fractional parts of a whole object to grasp the concept of fractions fully. Lortie-Forgues et al. (2015) found those activities that required representing, comparing, ordering, and locating fractions on number lines improved knowledge of fraction values. Other methods involve comparing two-dimensional drawings of circles or rectangles; however, Strother et al. (2016) found that unit lengths are more easily compared on bar models and number lines. Additionally, the whole unit of one is more easily recognized than when using set models.

12) Working knowledge of place value is vital to understanding decimals. If a student cannot sight-read numbers accurately with self-assurance, it will be challenging to convert decimal numbers to equivalent fractions and percentages. While some students can automatically relate decimal numbers to money, specifically dollars and cents, others will not make that connection and thus struggle moving forward with operations involving decimal notation. Computerized interventions involving repetitive practice were studied by Lortie-Forgues et al. (2015) and shown to be beneficial in decimal arithmetic. Finally, the order in which these three mathematical concepts are taught effected arithmetic performance. Presenting percentages first, decimals second, and fractions third produced the best results.

13) The realm of “learning disability” in math: In some English-speaking countries, there is a label or category or exceptionality called a “learning disability“ in arithmetic. In general terms, this is a student who is average to above average in intelligence, yet for whatever reason, there is a significant discrepancy between their intelligence and achievement in math. So, for example, a student in the 8th grade who can only perform

4th-grade math may be considered “learning disabled“ in math in some places. There is an extensive process that rules out any vision or hearing or lack of instruction, or any other factor or variable which could be causing the child’s difficulty in math. Needless to say, the student may have some apprehension about math and “doing math.”

14) Understanding “word problems”: Word problems present their own set of challenges. Asking a child to add 4 plus 7 can result in the correct answer of 11. Asking the same child to solve a problem such as “Billy has 4 marbles, and his friend gives him 7 more, how many does he have altogether?” may result in consternation and apprehension. It could be due to the child not having a very good attention span or simply being overwhelmed when confronted with something with which they are not familiar. This becomes even more problematic when algebra comes along and the student is given a problem such as “9x=81, what does x equal?” There is a specific language that a student needs to know to grasp certain problems and also procedural knowledge, as in quadratic equations.

15) Children in the upper elementary grades are about eleven years old when the concept of fractions is introduced formally in public education. At this age, children are between the concrete and formal operations stages of development, according to Piaget’s stages of cognitive development as reported by McLeod, S. A. (2018). To fully grasp the concept of fractions, children must move beyond the concrete stage to compare, combine, and classify fractions. As these initial rational number concepts are often the child’s first experience with higher-order thinking skills involving mathematics, learning may be challenging and require repeated practice and encouragement. This notion is in line with Mental Contrasting with Implementation Intentions, MCII, a metacognitive intervention that was studied by Corno and Anderman (2015). The MCII technique permits children to write down positive thoughts about a task, along with obstacles associated with the task, in the form of “If…(obstacle), then I will…(respond to overcome or work around the obstacle)” statements. This technique, and others like it, may prove to be useful tools to maximize student ability and success while managing performance anxiety.

16) Fractions, decimals, and percentages are challenging to teach and learn. If the instructor has limited knowledge of understanding rational number arithmetic operations, the students will not comprehend the concept either. When too much emphasis is placed on memorizing techniques instead of understanding the mathematical principles associated with those techniques, students' performance suffers. Lortie-Forgues et al. (2015) found that minimal instruction in fraction division resulted in a lower mastery of mathematical performance. One main reason for not spending much time on fraction division was linked to textbook contents.

17) Another link to mathematical performance was textbook explanations of mathematical operations. Imagine a scenario in which teachers with insufficient mathematical training use a textbook with inadequate or insufficient explanations of arithmetic concepts with which to teach their students. Some texts may have sound explanations for a given concept but lack alternative methods for explaining that concept. In addition, if students do not have sufficient mathematical abilities with whole numbers, the concepts involved with fractions become increasingly demanding. Not all students will learn from the same manner of teaching. Variety in the presentation may be essential to understanding these specific mathematical principles.

18) Fractions, decimals, and percentages play a crucial role in students' feelings about mathematics. Often, if a student struggles with learning fractions, they develop a dislike for mathematics. Fractions, decimals, and percentages are elementary to school math and daily life. We use fractions, decimals, and percentages to describe many situations in everyday life, such as cooking, interpreting distance on maps, defining grades, monetary transactions, calculating discounts, and more. There is also a connection between understanding fractions and having success in algebra.

19) Motivation: Teachers of mathematics, algebra, geometry, trigonometry, calculus, and numerical analysis must pay particular attention to motivation. They need to motivate students to want to learn math. They need to be enthusiastic, energetic, and excited about teaching math. They need to motivate students to want to learn, keep them motivated when confronted with difficulties, and enhance their motivation toward the end of the school year. This approach is not easy and requires a certain commitment on the part of the teacher.

20) Sullo (2007) has written an excellent book entitled "Activating the Desire to learn." He indicates that teachers need to understand internal motivation and reflect on both competition as well as cooperation. We need to assist the student to value learning and help them self-evaluate. There needs to be a "need satisfying environment" in the school, and teachers need to foster positive relationships with students.

21) Organization and the Organization of Number facts: Garnett (1992) developed the idea of instructional sequence, and this writer will attempt to provide some models, examples, or templars. There is the +1 principle- and for example, the teacher teaches 2+1= and then 3+1= and then 4+1= and continues in this manner. There is the +0 principle wherein 2+0=, then 3+0=, and so forth. There is the "ties" instructional sequence, and here are some examples: 4+4= then 5+5= and so on. Another sequence is "Ties+1," and in this, we see. 4+5=, 5+6=, 6+7=, and one can follow with a "Ties+2" approach- 4+6=, 5+7= , 6+8=, and so on.

The plus ten number facts can be seen herein: 1+10=, 2+10=, 3+10=, and so on.

The plus 9 number facts can be seen in this model: 6+9= is one less than 6+10=, and so on.

This pattern can be continued henceforth: 2+5=, 2+6=, 2+7=, 2+8=, 2+9=. And then, 3+6=, 3+7=, 3+8= and then 4+7= and 4+8=, and 5+8 and so forth and so on.

Thornton and Toohey (1985) developed a slightly different methodology. First, there are "count ons"- Herein, there are numbers that are simply added- one can add 1,2,3 and see the increase in the numbers. They have the +0 principle- 2+0=2, 3+0=3, and 4+0=4, and whenever one adds a zero- there is no appreciable or significant change in the number. Doubles allow students to see what transpires when a number is added to itself—SO—5+5=, 6+6=, 7+7=. Students in this model learn what numbers can result in a "10 sum"- so 7+3=10, 4+6=10, and so forth. This can be followed by 9's and other numbers. Thornton and Toohey include "near doubles," which may be more difficult for children to understand-

4+5=9 and 3+4=7. And students may hopefully grasp the slight increase in amount.

22) One extremely important educational psychology principle that applies to just about all learning is the massed versus distributed practice issue. In distributed practice, the teacher spaces the learning across the entire day for repetition and repeating as well as the week and allows for homework to practice any skill over time. This approach is better than attempting to teach something for an extended time on Friday for several hours and expecting the students to be able to remember and retain the information the following Monday. Schutte, Duhon Solomon, Poncy, Moore & Story (2015) conducted the most recent research on this, although the concept has been around for decades.

23) Technology: The hand-held calculator has probably revolutionized the teaching of math, and many of the hand-held calculator functions are now available on cell phones. Further, there are many “game-based” programs designed to remediate math skills, such as “Math Blaster” and many others. These programs utilize programmed instruction and assist the student in moving purposely thru the program until they reach a level wherein they need remediation or slow, purposeful, intense instruction. The program obviously rewards and reinforces the student for procedures using what is referred to in the field as “bells and whistles” to praise the student for successfully completing various aspects of math.

24) In some school settings, competent teachers can use advanced assessments such as the Key Math - to pinpoint, with a good deal of accuracy, the student’s weak areas. Such testing does consume a good deal of time but is often worth the investment for a struggling student.

25) Homework: Homework is perhaps a nasty word in some instances as it conjures up images of students working into the evening over quadratic equations or algebraic formulas. But, homework can and does serve a purpose- that of review and hopefully, if done correctly, can lead to mastery. Teachers need to look at the research on homework and perhaps configure a formula for how much is appropriate for each grade level and modify it as students become more and more successful.

26) Overall, while trying to learn math, students are also attempting to cope with developmental milestones and various psychosocial crises. Abraham Maslow, many years ago, indicated that we all have various basic needs, and for some students, they are attempting to get those basic needs met. These issues have been explored by Shaughnessy, Moffitt, and Cordova (2018). Teacher training programs often neglect the social and emotional needs of their students and are quite often ill-prepared to address those developmental and social needs. And in this age of school shootings and COVID-19, there is a pervasive sense of anxiety that permeates the schools and even the thoughts of parents. Teachers need to be sensitive to these issues and attempt to create a positive school climate that attempts to reassure students.

27) In this day and age of increasing demands on teachers, one is hesitant to mention one or two issues that arise again and again. Those issues revolve around the school day and school year. There has long been some debate that the schools in America need to extend the school day or school year. Obviously, this may need to be done in a prudent, judicious manner. But when one compares the U.S. to other nations around the world, one realizes that the U.S. has among the shortest school days and school years in the world.

28) Ongoing consultation and training as well as supervision: Teachers do not simply leave a teacher training program with all the skills, methods, and materials needed to be competent teachers for children of various races, religions, creeds, and cultures. Teachers need to be provided with time to discuss difficult students, difficult parents, and difficult situations. Obviously, teachers need time for lesson plans, and currently, many teachers are having to teach online via Zoom or Skype or some other platform -a task that has challenged many teachers- as well as many parents who are endeavoring to get their children to “log on” for their daily math lesson.

29) There has been a recent issue of a journal specifically devoted to the teaching of math. An overview can be found at: https://www.mdpi.com/journal/ejihpe/special_issues/RAME.

30) Few researchers have investigated the impact of the COVID-19 pandemic on the math skills of students. This influence is going to be an issue of concern in the near future as students return to school in the Fall.

Summary and Conclusions

This paper has cursorily attempted to review some of the main issues in mathematics instruction via a lens of educational psychology. Each one of the aforementioned issues could comprise a small chapter in a book, and interested readers are encouraged to delve into these issues keeping in mind their own situation (elementary, middle school, high school).

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