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Original Paper

Enhancing the Effectiveness of Feedback in Mathematics Teaching through Strategic Approaches

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Abstract

This article presents a methodological approach for updating feedback in school mathematics instruction. An analysis of psychological, pedagogical, and methodological sources reveals challenges related to insufficient attention given to the procedural and epistemological aspects of mathematical education. To address these challenges, Ramaprasad’s feedback model can be applied to define a student's position within the subject content axis. If the gap between the student’s current status and the established standards is addressed during feedback updates, it becomes possible to assess and correct the student’s cognitive and identification gaps. Criteria-based diagnostic tasks are proposed as a means to update feedback, supplementing traditional methods of diagnosing student performance in mathematics. These tasks focus on evaluating reasoning accuracy, comprehension of educational texts, and the mental processes involved in problem-solving. Such tasks advance feedback updating in the contextual, procedural, and conceptual domains. The article reviews various task types and outlines the criteria for their implementation. These tasks should be designed according to the specific characteristics of the mathematical content, the learning stage, and the student’s achievement level. The latter should align with one of the developmental zones: the zone of achieved development, the zone of proximal development, or the zone of current development. The goal of this article is to explore the didactic aspects of implementing feedback within the framework of Ramaprasad’s concept, and to develop methodological tools to enhance its effectiveness in the procedural diagnosis of students' educational and problem-solving activities in mathematics.

Keywords: Feedback, Teaching mathematics, Criteria-based mathematical tasks, Diagnostics, School students, Ramaprasad’s model

1. Introduction

It is widely recognized that providing adequate feedback during both current and final diagnostics of student learning quality is crucial for monitoring the effectiveness of the educational system and improving the learning process itself (Bouchey & Harter, 2005; Craven et al., 1991; Perevoshchikova, 2020; Rodionov, 2001, 2017). The pursuit of such improvements has led the global teaching community to implement national multi-level tests for students, offering achievement data at specific stages of particular curricula.

These tasks were initially designed to provide feedback on student performance, highlighting strengths and weaknesses to improve learning outcomes. A test task should theoretically function as an indicator to both identify and address learning challenges. How the results are analyzed and reported plays a crucial role in enhancing various forms of educational effectiveness (Baadte & Schnotz, 2014; Bloom & Hautaluoma, 1987; Bouchey & Harter, 2005; Lechermeier & Fassnacht, 2018; Nelson & Schunn, 2009; Perevoshchikova, 2020; Rodionov, 2001; Sarantsev, 2005; Zelina & Malakhova, 1987). This information serves as the foundation for both external (teacher diagnostics) and internal (student self-diagnostics) feedback, enabling necessary adjustments in the organization of the overall learning process for teachers and in educational and problem-solving activities for students.

An analysis of psychological, pedagogical, and methodological sources reveals various approaches to implementing feedback strategies, particularly in mathematics teaching. These include: the personally-oriented approach, which emphasizes creative activity and the emotional and value-based attitudes toward various aspects of the world (Ganeev, 1997; Nilson, 2003; Simonsmeier et al., 2020); the technological approach, which focuses on automated learning (Rodionov, 2017; Rodionov et al., 2020; Tseng & Tsai, 2007); the task-based approach, which views the learning process as a system of solving problems by both the subjects and objects of the learning process (Bouchey & Harter, 2005; Costello & Crane, 2013; Pólya, 2002); the reflexive approach, which addresses the relationship between external and internal diagnostics of subject-related preparation (Craven et al., 1991; Ryzhik, 1995; Vancouver & Tischner, 2004); and the professional pedagogical approach, which focuses on preparing future and current teachers to organize effective feedback in the educational process (Bahri Yusoff, 2013; Dempsey et al., 2009; Sarantsev, 2005; Stepanov, 1991; Perevoshchikova, 2020).

Despite the significant contributions of these scholars in addressing the challenge of organizing effective feedback in education, a gap remains in the understanding of the feedback process. While most psychological and pedagogical studies examine various aspects of feedback, particularly in relation to students' search activities, methodological studies mainly focus on feedback organization based on traditional normative systems for diagnosing the subject competencies of school students. Moreover, many educational and methodological solutions provide limited data on intermediate student activities, hindering timely corrections of potential issues. This is partly due to the insufficient development of systematic feedback approaches in managing mathematics instruction, the use of one-dimensional methodological tools that offer feedback mainly for diagnosing knowledge and competencies, and a primary focus on final learning outcomes (often neglecting the procedure of knowledge assimilation during the learning process). Additionally, norm-referenced assessment systems, prevalent in schools (such as entry tests, ongoing tests, transfer exams, Basic State Exam, and Unified State Exam), often foster a sense of learned helplessness in students.

To address the aforementioned deficiencies, scaffolding could be a viable solution, as it emphasizes three key points in learning: determining the student's current stage of development, establishing feasible developmental paths, and identifying the didactic tools necessary to support such development (Anghileri, 2006; Blinko, 2004; Boulet, 2007; Denhere et al., 2013; Holton & Clarke, 2006; Livingston, 1997; Schoenfeld, 2002; Wiliam, 2007; Wood et al., 1976). According to Ramaprasad (1983), Professor Emeritus of Information and Decision Sciences at the University of Illinois (Chicago, USA), the concept of scaffolding must take into account the active role of students, as opposed to the traditional model where the teacher is solely responsible for implementing the specified functionality. While teachers are responsible for designing and creating an effective learning environment, students play an essential role in improving that environment. Both parties must do everything possible to mitigate the consequences of any failures. In this context, teachers, students, and peers who are invested in accurately assessing their subject achievements become the initiators of feedback implementation.

The proposed approach is based on recognizing the didactic features of feedback implementation while diagnosing the process of students' educational and problem-solving activities within the framework of Ramaprasad's feedback concept, as well as advancing effective techniques for achieving successful outcomes.

2. Methodology

Ramaprasad's concept is based on the instructional scaffolding strategy introduced by Wood et al. (1976) and is closely related to Vygotsky's (1978a, 1978b) zone of proximal development (ZPD) theory.

Vygotsky's theory offers a theoretical framework for understanding the teacher's role in learning. According to Denhere et al. (2013), this theory emphasizes the importance of keeping learners within their ZPD for as long as possible by providing engaging and meaningful tasks that are more challenging than what the learners could accomplish independently. In this context, the teacher acts as a key mediator in the instructional process. Both the teacher and the learners continuously require multilateral feedback to establish a common starting point for productive interaction.

Adequate data on students' zones of current and proximal development can provide teachers with a foundation for designing strategies that help students progressively reach their full potential.

In any given mathematics curriculum, the outcomes set for learners will not always align with their ZPD. For many students, these outcomes may be unattainable even with teacher support, and attempts to cover all curriculum objectives may leave a significant number of students behind in mathematics. As such, the importance of evidence-based feedback in addressing unsatisfactory performance in mathematics becomes particularly critical.

A key challenge in implementing Vygotsky's concept in the educational process is that teachers often struggle to identify the unique ZPD of each student. To address this challenge, Ramaprasad proposed a three-stage model (Fig. 1), where the teacher uses curriculum outcomes as targets and works to identify learners' deficits in relation to these targets (Wiliam, 2007).

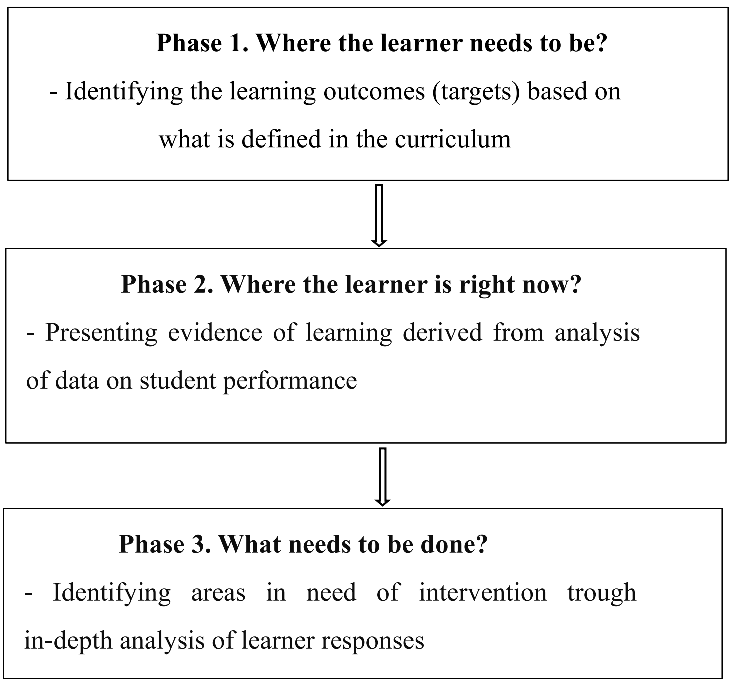

Figure 1. Ramaprasad's model

Although linear in structure, Ramaprasad's model supports Vygotsky's framework, with the distinction that it places limitations on where the learner needs to be, based on curriculum outcomes. One way to interpret Ramaprasad's model is to view the learner's ZPD as the gap between what they should know (as defined in the curriculum) and what they actually know. As a result, learning targets become more defined for the teacher, who must now identify learner deficits in relation to these curriculum goals.

A key feature of Ramaprasad's model is the role of feedback in determining the learner's current position (Phase 2 in Fig. 1). This feedback is essential for improving instruction and, more importantly, provides the evidence teachers need to work within the learner's ZPD (Wiliam, 2007).

On a broader scale, performance data will also be valuable to administrators, allowing them to make informed decisions when planning interventions to address student deficiencies.

Once curriculum targets are established (Phase 1), teachers must determine what their students already know (Phase 2) in order to plan instruction that aligns with learner needs (Phase 3). Detailed analysis of data from large-scale assessments, performed by experts, can provide teachers with the necessary feedback in Phase 2. This feedback will help classroom teachers target instruction to meet specific learning needs.

A major challenge in this process is identifying the nature of mastering specific mathematical skills.

Survey results have shown that many teachers give insufficient attention to this aspect, often placing full trust in testing materials developed by methodologists without conducting the necessary content analysis. As a result, the effectiveness of comprehensive feedback in mathematics instruction is compromised. This issue hinders the identification of students' understanding of the underlying semantic concepts of the mathematical content being studied.

3. Results

Based on the above, it is advisable to use criteria-based tasks as the primary means for implementing current feedback. These tasks provide objective diagnostics of the peculiarities in students' assimilation of educational actions and allow for the prediction of further progress (Dempsey et al., 2009; Perevoshchikova, 2000; Rodionov, 2017; Rodionov et al., 2020).

In this context, we define a learning task as one where the solution indicator is the independent mastery of the corresponding criteria-based actions by students. Criteria-based tasks should focus on assessing the assimilation of these actions, which form a set of operations that potentially ensure the fulfillment of the related learning objectives.

In accordance with Ramaprasad’s concept of feedback, criteria-based tasks should be considered within three domains: contextual (I), procedural (II), and conceptual (III) (Holton & Clarke, 2006; Rittle-Johnson & Koedinger, 2005).

I. Contextual domain: This refers to students’ competence in applying knowledge to specific real-world situations, developed through ordinary informal interactions with the world. For example, this domain is relevant when solving word problems in primary school or applied tasks in high school. The results of the Program for International Student Assessment (PISA) test have shown that such problems add further complexities to the implementation of mathematical problem-solving activities. These difficulties are related to the specifics of pseudo-real and real content tasks, such as updating initially hidden relationships between the components of a problem, assessing the sufficiency, consistency, and redundancy of these components, and searching for possible interpretations of a set of such relationships.

Tasks that involve formulating main and additional questions to the problem, identifying anomalies in the conditions, and translating the problem into an alternative mathematical language (e.g., constructing a diagram) should be used as criteria-based tasks. These tasks allow for the current assessment of students’ relevant competencies. In solving such tasks, students naturally engage in the structural analysis of the problem by identifying hidden connections between its components (Perevoshchikova, 2000; Rodionov, 2001; Rodionov & Velmisova, 2008; Rodionov & Akimova, 2015; Ryzhik, 1995; Sarantsev, 2005; Stepanov, 1991; Zelina & Malakhova, 1987).

An example of a criteria-based task for formulating requirements for a potential problem situation, specified by a ready-made drawing of the original geometric configuration, is the function of a basic visual-figurative model of this situation (Rodionov, 2001, pp. 37–38): “Based on the examination of a triangle configuration, given together with its midline (Fig. 2), identify all possible characteristic regularities thereof.”

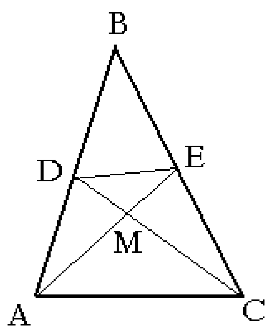

Figure 2. The triangle configuration

As a result, the following range of statements could be formulated:

a) The midline of a triangle divides it into a triangle and a trapezoid.

b) The diagonals of the resulting trapezoid are the medians of the original triangle.

c) Each median divides the triangle into two equal triangles.

d) The triangles DMA and EMC are congruent.

e) The triangle AMC and the quadrilateral DBEM are equal, etc.

The more relationships students can identify, verify, and justify, the higher their level of competence in the contextual domain. Similar tasks could be based on the analysis of carefully selected text problems (Zelina & Malakhova, 1987, p. 87).

Here is an example of an Isaac Newton puzzle:

"Grass in a lawn grows equally thick and at a uniform rate. It takes 24 days for 70 cows and 60 days for 30 cows to eat all the grass."

Next, possible questions for the problem could be formulated as follows:

a) How many cows are needed to eat the grass in 100 days?

b) How many days will it take for 10 cows to eat all the grass?

c) What is the maximum number of cows that can graze on the lawn for a week?

d) What is the maximum number of cows that can be kept grazing on the lawn at all times?

Notably, in addition to the feedback function, such tasks can also play a significant motivational role by giving students the freedom to choose the requirements of the task. While solving these, additional questions may arise that are important contextually. For example, the number of cows might be fractional. How should the answer be formulated in that case?

II. Students' performance in the procedural domain relates to the process of solving a subjectively new search-type problem, which involves identifying a plan for its step-by-step solution (development). Such a problem, situated within the ZPD, could initially be modeled by a teacher (or with the help of more capable students, in the case of group work).

A key challenge for feedback implementation in diagnosing students' mathematical training is the insufficient awareness of the search process, which often results in skipping stages of the solution. This issue hinders proper analysis of the process. Therefore, for effective feedback, it is advisable to use procedural criteria-based tasks that allow the solution sequence to be unfolded as fully as possible, bringing potential hidden problems to the surface.

In this context, when performing diagnostic procedural tasks, it is important to ensure that the solution is carried out step by step. This helps diagnose the correctness of the search process, identify its breakpoints, and uncover the reasons for any errors (Perevoshchikova, 2000, pp. 283–284). Additionally, it is beneficial to ask students to justify their choice of method or solution technique.

Here is an example of a task involving the selection of the correct variant:

"Choose several equations for independent solution from a list of proposed equations, each of which is assigned a certain number of points, so that the total number of points is no less than four."

$x^2 - 22x + 25 = 2x^2 - 20x + 1$

(1 score)

$(x + 1)^2 = 3(x + 7)$

(1 score)

$(x + 4)^2 = 2(4x + 11)$

(1 score)

$11x + 26 = (x + 5)^2 + (x - 2)^2 - (x - 1)(x + 1)$

(3 scores)

$2(x - 2)(x + 2) = (x + 1)^2 - 3$

(2 scores)

Here is a more complex task: "Find the value of ***a*** for which the equation $(\boldsymbol{a} - 1)\, x^2 + \frac{1}{4}\boldsymbol{a} = -\boldsymbol{a}x$ has one root".

(5 scores)

The task is coupled with some questions that provide a procedural feedback:

- Are the proposed equations quadratic?

- How to determine the number of roots of a quadratic equation?

- What known methods for solving quadratic equations can be applied here?

- Which equations are advisable to solve using general solution? (Justify your answer);

- How can one determine the optimal way to solve an equation based on its type?

- Choose any unsolved equation and address it verbally based on the highlighted guidelines.

Notably, since students typically choose different equations that they solve in their preferred ways, it is possible to diagnose and self-diagnose both their procedural capabilities and their level of aspirations.

While solving mathematical tasks, school students may be required to update certain heuristic procedures. Such updating arises from the formation, encoding, recognition, and subsequent decoding of related intellectual patterns (schemes for the implementation of intellectual activity). These patterns create variable models for educational and research activity, serving as a kind of guideline for selecting the implementation vector.

Thus, the development process can be seen as a method for correlating the procedure of solving the initial problem structures with alternative analogues, implemented through current feedback updating ("looking back"). The logic of the search process according to this approach is illustrated in Fig. 3.

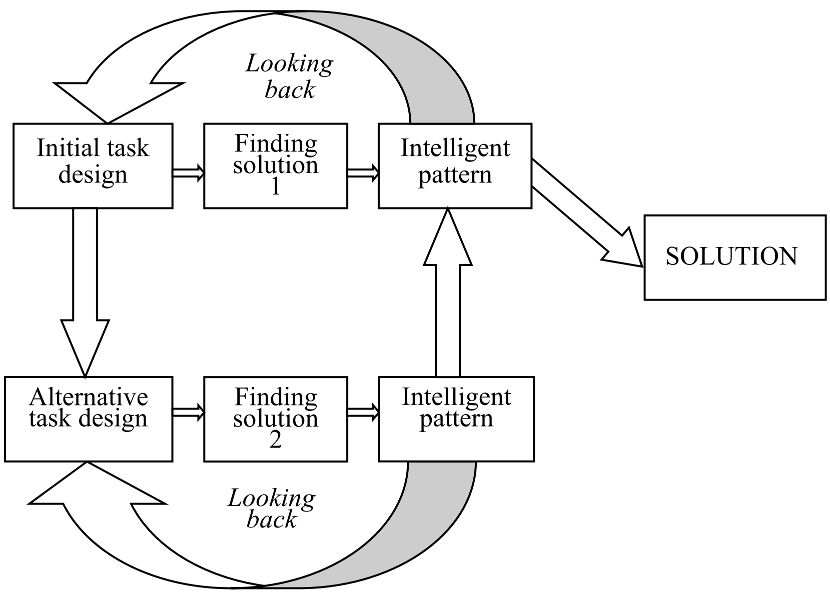

Figure 3. A logical sequence of the search process

It is demonstrated that feasible alternative means of solution are used when updating various model transformations of the problem conditions, thus ensuring the effectiveness of the search process (parameterization, specialization, limiting cases, reduction, alteration, etc.).

In general, updating a particular intellectual pattern is determined by the likelihood of linking the maximum number of elements of the task's subject area and the relationships connecting them to the task analysis. This is followed by the selection of the basic component of the initial configuration (Rodionov, 2001, 2017; Rodionov & Velmisova, 2008; Rittle-Johnson & Koedinger, 2005). Thus, updating various intellectual patterns that align with the mathematical constructions under consideration is one of the major procedural criteria.

For example, if the task involves a triangle with a median drawn, it would be advisable to draw a segment equal to this median from the middle of the side. This technique can then be applied to solving problems related to triangles, areas of plane figures, geometric inequalities, and so on.

In accordance with previous considerations, a student's readiness in the procedural domain can be assessed using criteria-based tasks that involve intellectual patterns, based on the corresponding prompts. For instance, when solving stereometry problems, it is often (though not always) helpful to consider planar analogues of spatial configurations, where certain intellectual patterns become more clearly visible (Sarantsev, 2005, pp. 97–98).

Here is an example of a criteria-based task: “The intersection point of the bisector planes of the dihedral angles of a tetrahedron is connected to the vertices of the tetrahedron. Prove that the volumes of the resulting tetrahedra are proportional to the areas of the corresponding faces of the given tetrahedron.” The corresponding plane analogue is provided: “The intersection point of the triangle bisectors is connected to the vertices. Prove that the areas of the resulting triangles are proportional to the corresponding sides of the given triangle.”

Evidently, considering a planar analogue of the problem is key to its solution. In this case, feedback can be updated when diagnosing both the content and the general developmental (construction of a planar analogue) components of students' mathematical training. Successful completion of such tasks by a student indicates a sufficiently high level of procedural competence.

III. Students' performance in the conceptual domain* assesses their ability to apply integrated knowledge of key mathematical principles and patterns (e.g., knowledge of numerical quantities, polygons, etc.). This knowledge should be understood as generalized knowledge—comprising principal ideas and methods—that organizes and encapsulates numerous specific mathematical facts, patterns, and solution methods in a hidden form. It should not be considered as isolated, independent units of mathematical content. In other words, conceptual knowledge forms the foundation for understanding the role of a specific problem within the broader context of the course, enabling the generation of new strategies or the adaptation of existing strategies to solve novel or more general problems.

It is advisable to assess students' readiness to generalize specific mathematical facts and, conversely, to relate them to more general mathematical laws as one of the main criteria for mastering conceptual knowledge. This underpins the selection of criteria-based tasks.

Here are some examples of criteria-based diagnostic tasks that evaluate students' readiness to assess a particular task situation in terms of a more general pattern with a conceptual character.

Task 1 (Stepanov, 1991, p. 49):

Prove that if $f(x, y)=f(-x,-y)$, then the graph of the equation $f(x,y)=0$ is symmetric about the origin. Check the truth for the graphs of the equations:

1) $x^2 + y^2 = r^2$;

2) $x^2 - y^2 = 0$.

The following task specifies diagnostics of a feasible reverse transition, when a student could demonstrate the readiness to generalize a particular pattern through a transition to a wider area of the elements for the subject area of the problem.

Task 2 (Pólya, 2002, p. 13):

a) Prove the following theorem: Let some point lie in the interior of an equilateral triangle and be located at a distance x, y, z from its three sides respectively; h is the altitude of a triangle. Then:

$$x + y + z = h$$

b) Generalize the theorem so that it is applicable to any point in the plane.

c) Establish and prove a similar theorem for stereometry regarding the distances from an interior point to the four faces of a regular tetrahedron.

d) Generalize the latter theorem so that it is applicable to any point in space.

The selection of such criteria-based diagnostic tasks, supplementing the traditional diagnostic apparatus in real educational practice, involves considering both the peculiarities of the material content and the students' mathematical development level.

4. Discussion

Efficient feedback implementation in students' mathematical activities is closely aligned with specific outcomes related to mathematical competence and the corresponding cognitive level. In this context, the following student achievement levels based on the scaffolding strategy can be distinguished:

I. Competent: Students can perform tasks independently, without any external support, including that of the teacher.

II. Partially competent: Students possess some skills to perform tasks with support, including the teacher's assistance.

III. Incompetent: Students lack the necessary knowledge and experience to perform tasks even with support, including that of the teacher.

The first student achievement level (I) corresponds to the zone of achieved development (ZAD), which helps teachers identify prior knowledge or a starting point for further instruction.

The second student achievement level (II) corresponds to the zone of proximal development (ZPD), indicating areas where teachers can help improve relevant student skills.

The third student achievement level (III) corresponds to the zone of current development (ZCD), representing curriculum requirements that are beyond the students' capabilities and require external intervention to address.

Criteria-based diagnostics for various scaffolding learning strategies in teaching mathematics, corresponding to the three student achievement levels, are presented in Table 1.

Table 1. Criteria-based diagnostics for various scaffolding types

Student achievement levels	**Scaffolding types**		
	Contextual scaffolding	**Conceptual scaffolding**	**Procedural scaffolding**
I. ZAD	Rigid orientation towards an external prescription when solving a given specific problem	Functioning in a strictly prescribed range of choice for the mathematical content elements	Almost uncontrolled chaotic or systematic enumeration of alternatives
II. ZPD	Updating of latent relationships between the subject area components of the problem, and appropriate interpretation and formalization thereof	Readiness to independently vary the studied content attributes, and search for various model representations of the studied material	Selection and conscious implementation of the optimal method for solving the problem, and reflection on the solution obtained

III. ZCD	Implementation of heuristic procedures based on targeted modification of the problem domain components	Focusing on a generalized model for the considered problem situations	Updating of intellectual patterns typical for solving thematic-related problems along with empirical proof thereof

The presented diagnostics establish a benchmark for constructing and applying criteria-based tasks at various stages of training and phases of the search process. In alignment with Ramaprasad's model, this approach involves considering the planned outcomes of mathematics teaching at a specified level, the current results of students, and the feasibility of addressing any deficits in student competence.

5. Conclusions

It is evident that the nature of adequate feedback provided to teachers and other stakeholders in diagnosing the outcomes of mathematics teaching, plays a crucial role in enhancing mathematical education. An analysis of psychological and pedagogical literature, as well as observations of the educational process, has highlighted significant challenges. These challenges arise from the current diagnostic systems focusing primarily on the problem result rather than the solution process. Consequently, there is a need to establish the conceptual foundations for implementing current feedback and diagnostic tools that can ensure its effectiveness.

To address this issue, it is proposed to utilize Ramaprasad's three-stage model with feedback, where determining a student's current position on the competency scale is key. This position is then compared with the curriculum objectives to adjust the training process according to the students' needs and the requirements set by the federal state educational standards.

Ramaprasad's model is effective in providing feedback during the process of diagnosing the procedural aspects of mathematical skill implementation. Feedback, in this context, determines the level of students' awareness of the relevant subject content. Such diagnostics can be carried out using criteria-based tasks that incorporate procedural characteristics of educational and search activities.

Criteria-based tasks, based on the scaffolding concept, should be considered within three domains: contextual, procedural, and conceptual. The essential features and examples of tasks relevant to each domain, along with criteria for their implementation, have been specified. These tasks are selected according to Ramaprasad's concept of feedback, taking into account both the specific mathematical content and the students' achievement level in mathematics.

The three achievement levels based on students' mastery of mathematical skills have been defined and clarified: the zone of achieved development (ZAD), the zone of proximal development (ZPD), and the zone of current development (ZCD). The content of each level, in relation to various scaffolding types, has been outlined to provide a benchmark for selecting criteria-based tasks at different stages of training and phases of the search process.

The criteria-based tasks discussed are not intended to replace traditional methods for diagnosing students' mathematical training. Instead, they supplement these methods by enhancing feedback capabilities when assessing the assimilation of cognitive tools (such as the correctness of reasoning, level of educational text comprehension, and features of mental search implementation).

Future research will focus on exploring the possibilities of providing structured, reversible online interaction among the participants in the educational process. Such interaction could serve as an effective tool for the timely support of the educational and search process. This interaction may be organized through adaptive technology for teaching mathematics, as outlined in previous publications (Rodionov, 2017; Rodionov et al., 2020).

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