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Original Paper

An Episode in Analytic Reasoning: Who Really Discovered the Mass-Energy Equivalence?

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Abstract

In the early 1950s, the answer to the question "Who invented the equation $\mathrm{E} = \mathrm{mc}^2$?" which was thought to be established beyond any doubt, was seriously contested, when in 1952, Herbert Ives asserted that it was Planck and not Einstein who actually first proved the equivalence of mass and energy. After a brief period of calm, the interest in the question was revived in late 1970s and early 1980s, this time with most of the articles written on the topic being in support of Einstein. The dispute persists to this day.

In this paper, using both historical facts and basic laws of logic as it applies to proofs, I will argue that perhaps this is a false dichotomy or an artificial controversy. It was, in my opinion, the accumulation of the shared knowledge rather than one luminary scientist that led to the formulation of a useful mass-energy relationship.

1. Introduction

In the early 1950s, a matter of historical precedence, one which up to that point had been one of the least controversial facts of history of sciences, namely, the identity of the architect of the arguably most famous equation of modern physics, $E = mc^2$, was resolutely disputed. In 1952 Herbert Ives (1882-1953), a well-known proponent of the Lorentzian ether theory, published an article in which he asserted that it was the German physicist and the originator of quantum theory Max Planck (1858-1947) and not Albert Einstein (1879-1955) who should be credited by being the first to establish the idea of the equivalence of mass and energy (Turner and Hazelett 1979). For, Ives alleged, Einstein's derivation in his seminal 1905 paper was logically flawed, in that it contained a circular argument (Ives, 1952; Lalli, 2013). The same idea was later iterated in some books (Jammer, 1961).

After remaining dormant for about a couple of decades, the interest in the question was revived in late 1970s and early 1980s, this time with most of the articles written on the topic being in support of Einstein (Miller, 1981; Stachel & Torretti, 1982; Fadner, 1988). The controversy seems to have subsided for now with no side being able to claim a clear victory.

In this paper, we will concentrate on some historical facts and basic laws of logic as it applies to proofs, and try to answer this question. Except some mathematical derivations, the paper deals mostly with some historical facts and assumes that the underlying physical theory is well-known. If needed, the reader may find the basic information on theory of relativity and related topics covered in this paper in Torretti (1983), Stachel (1989), d'Inverno (1992), Frankel (1997), Brown (2005), Rindler (2006), Freund (2008), Serway and Jewett (2018), and Günther et al. (2019).

2. Who Discovered the Equivalence of Mass and Energy?

The first fundamental issue here is to determine if there were any formulations of mass – energy or rather matter – energy equivalence prior to 1905, and if so, whether the radically momentous implications of this relation were fully comprehended and appreciated by its originator.

As early as 1880s, physicists, especially those exploring the electromagnetic theory of matter, began to notice that the electromagnetic energy generated by moving charged spheres had the effect of increasing the masses of these spheres. In 1881, the English physicist Joseph John Thompson (1856=1940) formulated this increase as

$$\Delta m = \frac{4}{15}\frac{\mu e^2}{r}$$

where μ stands for the magnetic permeability, e the charge of the electron, and r the radius of the sphere.

This was improved in 1889 to

$$\Delta m = \frac{2}{3}\frac{\mu e^2}{r}$$

by the English physicist and mathematician Oliver Heaviside (1860-1925).

In 1900, the French mathematician and physicist (and philosopher of science) Henri Poincaré (1854-1912) showed that electromagnetic energy had momentum and came up with the equation

$$\rho = \frac{J}{c^2}$$

relating the mass density ρ to the energy density J (Miller, 1981), and in essence, came up with an equation that was equivalent to $E = mc^2$. However, Poincaré interpreted his equation as one that affected the energy density of a fluid of radiation and not as one about *matter*. It was shown later that the related equation involving matter could be derived from Poincaré's analysis, but Poincaré himself did not do that (Poincaré, 1900).

Another important name associated with the theory was the Dutch physicist Hendrik Anton Lorentz (1853-1928). In 1892, Lorentz wrote a comprehensive paper on electromagnetic theory. In this paper, he started out with Maxwell's equations expressed in the ether (rest) frame S, and then applied Galilean transformations to write these equations in an inertial reference frame S'moving at velocity v with respect to S. Time was not transformed and was taken to be the same in both frames.

Lorentz assumed that objects were at rest in S. However, the equations in S were not the proper form of the wave equations. To this end, Lorentz devised a second coordinate system S'' that would yield the proper equations. In this system, he obtained reasonable agreement with experimental results by solving the wave equations to second order in $\frac{v}{c}$.

Later the same year, Lorentz used these equations to propose an explanation for the Michelson - Morley experiment. By the end of nineteenth century, prevailing theories were that there had to be an absolute reference frame (the ether) with respect to which the rest of the universe was stationary. At the time, there was no experimental verification of the ether theory and physicists were trying to design experiments to garner empirical evidence in support of this hypothesis. One such experiment, and possibly the most illustrious one, was performed by Albert Michelson (1852-1931) and Edward Morley (1838-1923) in 1887. The basic idea was this: light would sometimes travel in the same direction as the ether, and others times in the opposite direction. Thus, if one could measure the speed of light in different directions, one would be able to determine the speed of the ether relative to Earth, establishing, consequently, its existence.

Michelson and Morley were able to measure the speed of light by looking for interference fringes between the light which had passed through the two perpendicular arms of their apparatus, the Michelson interferometer, specifically built for this purpose. They found no discernible fringes that would indicate a different speed in any orientation.

In 1895, Lorentz concluded that the null result obtained by Michelson and Morley was caused by the effect of contraction made by the ether on their apparatus and introduced the length contraction equation

$$x = \frac{x'}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Although the interpretation was incorrect, the equation itself turned out to be the one used later by Poincaré, by Lorentz himself, and eventually by Einstein to establish the space-time contraction and to eradicate the notion of ether permanently.

In 1899, and again in his paper *Electromagnetic phenomena in a system moving with any velocity smaller than that of light* (1904), Lorentz added time dilation to his transformations, namely the concept of *local time*

$$t = \frac{t' - \frac{vx'}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

and came up with the matrix

$$\begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \gamma & \theta & 0 & 0 \\ \theta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix}$$

where

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}$$

and

$$\theta = -\gamma \frac{v}{c}$$

In 1905, Poincaré referred to this matrix as the *Lorentz transformation.*

In 1904 Lorentz postulated that his coordinate substitutions represented physical transformations and showed that distortion in the size of a spherical charged particle was in accordance with these transformations. In other words, he obtained the correct equations for relativity, including mass as a function of velocity.

Meanwhile in 1902 and 1903, the German physicists Walter Kaufmann (1871-1947) experimentally showed that the mass of an electron increased with increased velocity.

The first obvious statement about the equivalence of mass and energy was published in 1904. In his book, *Radioactivity: An Elementary Treatise*, the English radiochemist Frederick Soddy (1877-1956) claimed that one should not expect the law of conservation of mass to hold true for radioactive phenomena. He cited Kaufmann’s experiments, and maintained that atomic mass had to be regarded as a function of internal energy, and that as the latter dissipated, so would the former (Soddy, 1904).

Also in 1904, the Austrian physicist Friedrich Hasenöhrl (1874-1915) derived the equation of mass increase of a moving cavity containing electromagnetic energy E as

$$\Delta m = \frac{8}{3}\frac{E}{c^2}$$

(Hasenöhrl, 1904) and later the following year, corrected it to (Hasenöhrl, 1905)

$$\Delta m = \frac{4}{3}\frac{E}{c^2}$$

These were certainly mass – energy equations; however, Hasenöhrl did not explore the general implications of these relations.

Thus, we come to the *annus mirabilis*, 1905. In his classical 1905 paper, *Zur Elektrodynamik bewegter Körper* (On the Electrodynamics of Moving Bodies) Einstein postulated that

(i) The speed of light was a universal constant,

(ii) The laws of physics were the same in all inertial frames.

From these assumptions, without any reference to the structure, size, or shape of the particles, Einstein obtained Lorentz equations, the mass increase equations, and a plethora of other results.

A few months later, Einstein published his famous paper, *Does the Inertia of a Body Depend upon Its Energy Content?* (Einstein, 1905). It was in this paper that the matter – energy equivalence first appeared in full generality: If a body gives off energy E in form of radiation, its mass diminishes by a factor of $\frac{E}{c^2}$.

Einstein continued his work on mass – energy relation in a 1906 paper, which contained the famous *photon in a box* thought experiment. Suppose a light of energy ΔE is emitted from one end A of a hollow cylinder and is absorbed by the other end B. According to laws of classical physics as applied to the momentum of radiation, the center of mass of the cylinder would shift. Suppose a massless carrier transports the same energy ΔE back to A. Initial position of the center of mass would have to be restored, but the cylinder would have moved on its own with no other net changes. To avoid this absurdity, one had to assume that there would also be a mass change of

$$\Delta m = \frac{\Delta E}{c^2}$$

in other words, the masses of A and B would change during the emission and absorption events.

In a 1907 paper, *The Inertia of Energy, as Demanded by the Principle of Relativity*, Einstein extended the concept of inertia of energy and obtained the relations

$$E = \frac{\mu_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

and

$$\mu_0 = \frac{E_0}{c^2}$$

where μ_0 denoted the rest mass and E_0 the energy measured in the rest frame. These formulas were very significant developments in physics for they were the first correct equations relating rest mass and rest energy. The concept of *rest energy* was the most revolutionary idea of this entire subject.

The equivalence of the relationships $E = \frac{\mu_0 c^2}{\sqrt{1-\frac{v^2}{c^2}}}$ and $E = mc^2$ depends on defining force as the rate of change of momentum, p. This would imply

$$p = \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}}$$

and consequently,

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

In 1907 Max Planck wrote a paper on relativity theory, *The dynamics of motion systems*, in which, using Lorentz transformations, he derived several results including

$$\Delta m = \frac{\Delta E}{c^2}$$

for heat energy. In the same paper, Planck stated that Einstein had obtained the same result in 1905, but there was a flaw in his argument (Planck, 1907). In fact, this remark became the essence of Ives's argument that priority should be assigned to Planck and to Einstein.

There are several reasons why most physicists give priority to Einstein. First of all, as we have seen, most of the prior derivations did not have the correct relativistic basis and at none addressed the applications of the relation $E = mc^3$ to rest energy. Secondly, prior to 1905, none of the physicists that dealt with this equation in some form or another referred to "matter;" most derivations concerned fictitious fluids of radiation. Moreover, not being a member of academia, Einstein was, in all likelihood, unfamiliar with the academic discourse of the period (Isaacson, 2008).

In the next section I will look into the validity of Ives' argument that Einstein's derivation was logically flawed.

3. Einstein's Derivation of $E = mc^2$

Suppose an object is at rest in the (x, y, z) coordinate system (the rest frame). Suppose, moreover, another frame (x', y', z') is moving with velocity v with respect to the rest frame. The event, which can be observed from both frames, will be the emission of some electromagnetic energy. Throughout the subscript i will be used to denote the initial state and the subscript f to denote the final state.

Suppose, before the event, the object has energy E_i as observed by an observer in the (x, y, z) frame and $E_i{}'$ as observed by an observer in the (x', y', z') frame. Let now an electromagnetic energy of $L/_2$ be emitted at an angle φ and simultaneously in the opposite direction as measured in (x, y, z). The directions being opposite and the emissions being simultaneous imply that the object will remain at rest in the (x, y, z) frame. Let the energy of the object observed after the event be E_f in the (x, y, z) frame and $E_f{}'$ in the (x', y', z') frame.

Einstein argued as follows: we have from the principle of conservation of energy in the rest frame

$$E_i = E_f + L$$

and from the principle of conservation of energy in the moving frame

$$E_i' = E_f' + \frac{L}{\sqrt{1 - \frac{v^2}{c^2}}}$$

where the second equation was taken from his 1905 paper.

Then, Einstein argued, it was clear that

(1) $$E' - E = K + C$$

where E' was the object's energy measured in the moving frame, E the energy measured in the rest frame, K the kinetic energy of the object observed from the moving frame, and C an arbitrary additive constant. This was the equation to which Planck objected and later Ives claimed contained the result that was to be proved.

Let us try to see why Einstein might have written the above equation. In general, the total energy of an object is the sum of its kinetic energy, potential energy, internal energies, and an arbitrary constant. Consequently, in the rest frame one can write

$$E_r = K_r + V_r + I_r + C$$

and in the moving frame

$$E_m = K_m + V_m + I_m + C$$

where quantities with subscript r are measured in the rest frame, and those with subscript m in the moving frame. Subtracting the top equation from the bottom equation and noting that the object has no kinetic energy in its rest frame, we obtain

$$E_m - E_r = K_m + (V_m - V_r) + (I_m - I_r)$$

So, Einstein's assumption was that $(V_m - V_r)$ and $(I_m - I_r)$ could be written as an additive constant. Now, it is natural to assume that potential energies of the two systems would differ by an additive constant. The internal energies, which result from the movement of smaller particles making up the object, would involve a velocity dependent part, but that would be included in the kinetic energy part of the equation. From the point of view of classical mechanics, $(I_m - I_r)$ would be a constant. Consequently, Einstein's assumption was a classical, non-relativistic statement about kinetic energy.

Let us now see how Einstein concluded his derivation. From this equation Einstein obtained

$$K_i - K_f = \left(E_i' - E_f'\right) - \left(E_i - E_f\right)$$

and consequently,

$$K_i - K_f = L\left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1\right)$$

Now since

$$\frac{1}{\sqrt{1 + x}} = 1 - \frac{1}{2}x + \frac{3}{4}x^2 + \cdots$$

by ignoring terms that are squared or higher we get

$$\frac{1}{\sqrt{1 + x}} \cong 1 - \frac{1}{2}x$$

implying

$$K_i - K_f \cong \frac{1}{2}\frac{L}{c^2}v^2$$

On the other hand, since $K = \frac{1}{2}mv^2$, we have

$$\Delta K = \frac{1}{2}(\Delta m)v^2$$

Substituting this in $K_i - K_f \cong \frac{1}{2}\frac{L}{c^2}v^2$, we get

$$\Delta m = \frac{L}{c^2}$$

In his 1952 paper, Ives used a different derivation (Ives 1952). Starting out from the equation

$$K = mc^2\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1\right)$$

derived by Einstein in his relativity paper, Ives obtained

$$K_i - K_f = (m_i - m_f)c^2\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1\right)$$

Using the relations

$$K_i - K_f = (E_i' - E_f') - (E_i - E_f)$$

and

$$K_i - K_f = L\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1\right)$$

he obtained

$$(E_i' - E_f') - (E_i - E_f) = L\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1\right)$$

$$= (K_i - K_f)\frac{L}{(m_i - m_f)c^2}$$

which could be considered as the difference of the two equations

$$E_i' - E_i = \frac{L}{(m_i - m_f)c^2}(K_i + C)$$

and

$$E_f' - E_f = \frac{L}{(m_i - m_f)c^2}(K_f + C)$$

Ives argued that by assuming the validity of equation (1), Einstein was in fact arguing that

$$\frac{L}{(m_i - m_f)c^2} = 1$$

that is,

$$\Delta m = \frac{L}{c^2}$$

Those who criticize Ives use the argument that Einstein’s approach was *ad hoc* rather than circular; the postulates of a scientific theory always introduce the result. A conclusion can be obtained through a deductive argument if that conclusion is implicitly present in the postulates. In other words,

Suppose there are n necessary postulates in the form of mathematical statements in a system none of which are superfluous. Then it is possible to use any (n – 1*) of the postulates to show that the remaining postulate can be written to contain the conclusion that is to be proved.*

I agree with Ives as far as the circularity of Einstein’s original derivation is concerned. But, Einstein had derived the key to relation

$$K = mc^2\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1\right)$$

flawlessly, and in that sense, should still be credited as the discoverer of the equation $E = mc^2$ showing how mater and energy was related.

Indeed, it does not make sense to credit one person with the discovery of the fundamental relation $E = mc^2$. It was the shared knowledge and the historic accumulation of facts and observations that led to this relation. To disregard or to overlook the historic process of development and to designate one moment in time as “the time of discovery” is, from a pedagogical point of view, counterproductive to say the least. Question of priority, especially in this particular case, does not make sense – seeing how many other physicists had studied and attempted to describe the relationship between matter and energy at about the same time. The more interesting question might be why as scientists we always try to crown a single person with a scientific discovery. Science is not about competitions between some anointed minds, it is about cooperation and complementation between hard working scholars of all nationalities, genders, and age groups.

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