

Original Paper

All textbooks are not created equal: An international textbook study of multiplication and division

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Abstract

The paper reports on a study analyzing the differences in how the concepts of multiplication and division are depicted in primary school mathematics textbooks from three countries. This study contributes to the growing field of textbook analysis by focusing on an international comparison of curricula materials, specifically targeting younger grade levels and conceptual understanding of key concepts. Multiplication & division are important concepts for young learners of mathematics to understand because they provide the fluency skills and knowledge essential for higher level mathematics. The authors utilize content analysis to analyze three countries (United States, Poland, Mainland China) prominently used textbooks in childhood mathematics education classrooms. Results depict several major differences in curricula that may have correlations to a country's mathematics education outcomes. Findings suggest policy makers and education leaders focus on evaluating curricula materials for primary mathematics education.

Keywords: mathematics education, content analysis, international study, multiplication, division

Introduction

Since the widespread dissemination of the TIMSS (Trends in Mathematics and Science Study) and PISA (Program for International Student Assessment) in 1995 and 2000 respectively, countries began examining their mathematics education programs comparing them with higher scoring countries (Drew, 2011; OECD, 2010). In the United States, for instance, these findings have caused the enactment of several legislative policies (e.g. America Competes Act, 2007; Educate to Innovate, 2009), incited curriculum debates regarding what students should be learning, questions over the efficacy of state-run teacher training programs and their prescribed pedagogical doctrines, and even questions about the national education department's ability to serve the needs of a dynamically shifting societies across the world. According to international comparison studies such as TIMSS and PISA, United States students comparatively score mediocre at best while countries, like China and Singapore outperform children in comparatively similar modern countries (e.g. Fleischman et. al. 2010). The data in both the TIMSS and PISA studies have caused researchers and education policy makers across many countries to question their educational systems, curricula choices, and teacher preparation programs. The NAEP (national assessment for educational progress) 2022 report in the United States confirmed what many stakeholders and teachers feared - mathematics proficiency levels declined (and were already at alarmingly low rates) after the covid-19 pandemic.¹

Differences in mathematical proficiency can be found in early grades even before students take these international texts, raising questions about when educational reforms ought to take place (Lonnemann et al., 2019). In Poland, students' performance in Maths reduces proportionally to the number of years spent on learning it. This phenomenon is observed even at the first stage of formal education - between kindergarten and first grade. According to Edyta Gruszczyk-Kolczyńska (2021) more than 50% of Polish

¹ <https://www.nationsreportcard.gov/mathematics/states/scores/?grade=4>

six-to-seven-year olds present a high degree of aptitude in mathematical skills. However, after just a few months of education in class I, only one in eight students presents high mathematical skills. What are the causes of this phenomenon? First, early school math education is integrated with other educational areas: native language or nature, which makes mathematical problems less significant for students than the thematic areas illustrating them, such as animals. Other reasons include shortening the time for mathematical education, which is indirectly due to the belief of early school teachers about their low mathematical competences (and consequently fear of incorrect teaching of their students). Another reason for lowering both the interest and mathematical skills of students is the practice of paper mathematics (Gruszczyk-Kolczyńska, 2021), which means learning mathematics only on the basis of exercises in books, and thus the representation of iconic and symbolic, without reference to models, and, substitute collections or operating on subjects that are needed, especially by the younger students who develop their operational thinking.

There is a confluence of variables that influence a country's comparative mathematics test scores, many of which are not directly related to what happens in a classroom, such as the cultural, class, gender, and many other socio-political identity markers (e.g. Apple, 1992; Gabbard & Atkinson, 2007; Martin, 2008). Valuable contributions in this field have been made and continue to shed light regarding how systemic systems of oppression operate within an educational system affect certain groups of students disproportionately. As scholars of mathematics education, the authors of this paper want to acknowledge these larger inter relational structures and practices, yet assert that studying important variables, such as curricula choices, can offer important information needed to understand the disparities across countries' mathematical education systems. After all, contributions in ameliorating mathematics instruction should consider multiple lenses, asking specific questions, and offering varying analysis.

There are many studies reporting on the educational and cultural differences between Chinese, Japanese & Korean students to American students. (e.g. Lai & Wong, 2017; Huntsinger et al., 1998; Zhou, Peverly, & Lin, 2005), and some studies comparing European with Chinese kindergarteners (e.g. Lonnemann et al., 2019). Wang and Jin (2005) believe we should look into teaching practices to explain these differences, however, they also attribute much of the variation to parental beliefs regarding the value of effort. Guo et al. (2018) found that studying done outside of school hours and spaces is a strong indicator of PISA results. On the other hand, many researchers have found a correlation between teacher beliefs and their students' performance (e.g. Kleickmann, et. al. 2012). Additionally, teacher education and professional competence are markedly different across regions and countries (Blömeke, Kaiser, & Lehmann, 2010; Blömeke, Kaiser, Döhrmann, & Lehmann, 2010).

While there are certainly many other factors not specified above that may play a role in the disparities in international math test scores, such as language, culture, diversity of student population, racial and economic injustice, nevertheless it is still important to understand the factors that we may be able to control and change - located in the school system and its curriculum. One of these important factors is textbooks, which are referred to as prescribed or intended curriculum. Textbooks are the primary tool that are designed to guide and support learning in the classroom and instructional choices made by the teachers in the classroom. (e.g. O'Keeffe, 2013; Remillard, 2018) They are also artifacts that mediate the teaching and learning in schools, prescribing what and how content is taught and presented as well as what methods and/or conceptual models are left out. (e.g. Rezat, 2008) However, there is a clear dearth of research on the actual use of textbooks in mathematics education with several researchers pointing out that empirical research on mathematical texts is scarce (e.g. Love & Pimm, 1996)

To address these issues, educators and researchers have repeatedly called for mathematics education to be based on empirical evidence and conclusive research analysis as well as more comparative studies. In agreement with others, we contend, understanding how mathematical concepts and procedures are depicted in widespread curricula materials such as textbooks can help researchers understand at least one of the variables to account for the difference in international testing results as well as stagnated or declined proficiency test results. This study contributed to this field of inquiry by probing textbooks through a content analysis methodology to understand the ways in which multiplication and division concepts and processes are depicted in popular used textbooks. We chose to specifically look at multiplication and division since these ubiquitous mathematical operations are vital to understanding the base ten system and for future mathematics learning at the higher conceptual level. As we will explain in

the next section, multiplication and division provides learners with a depth of understanding numbers and their relationship and is the precursor for higher level thinking in mathematics. Ultimately, it is where mathematics becomes more abstract than may have been previously taught and for some learners, more difficult.

Our research question is:

- What are the differences and similarities between how multiplication and division concepts are introduced among international textbooks?

We began with a theoretical grounding in best practices of teaching multiplication and division. Next, we offer a review of the current literature in comparative textbook analysis. After this review, our data points, research question, and methodological framework is outlined. In the findings and discussion sections of this paper, we offer critical examination of our results. We end this paper with recommendations for mathematics education as well as considerations for future research agendas.

Review of the Literature on multiplication & Division

Multiplication and division are vitally important concepts for learners of mathematics to learn. For reasons not clearly understood, multiplication and division are much more complex of a concept than addition and subtraction (e.g. Dube & Robinson, 2018; Vula, & Berdynaj, 2011). Research in the acquisition of the concepts of multiplication and division are not addressed nearly as much as addition and subtraction. This is unfortunate since multiplication and division are viewed as necessary foundational knowledge for higher level mathematics (e.g. Findell, Swafford & Kilpatrick, 2001; Nunes, Bryant, & Watson, 2009). Nunes & Bryant (1996) studied how “multiplication and division represent a significant qualitative change in children’s thinking.” (cited in Vula & Berdynaj, 2011, 144) and are not simply the inverse of one another as is taught. Dubé & Robinson (2018) explain that students’ understanding of the relationship between division and multiplication is more difficult for students than understanding the relationship between addition and subtraction, yet little research has been put forth to understand this phenomenon. Thus, this area of mathematical content knowledge is an essential area of investigation into the differences between countries’ intended curriculum choices.

Izsák & Beckmann (2019) contend that there is a large body of research on topics related to the teaching of multiplication, yet it is a complex domain that is not easily taught to pre-service teachers nor understood by in-service teachers. Multiplication thinking is a crucial stage for children’s mathematical learning and sets the stage for the more abstract and complex concepts of higher-level mathematics. Hurst (2008) describes 2 stages of development in mathematical thinking, the additive stage and the multiplicative stage. First, “Adding numbers tells you how many things (or parts of things) you have when you combine collections. Then, multiplication is useful if you want to know the result of scaling some quantity” (cites Devlin, 2008, p. 1). Wright (2011) suggests that children need to reconceptualize their thinking about multiplication and division to understand the “multiplicative situation,” which refers to any scenario or problem where you need to combine equal groups to find a total. Squire and Bryant (2002) argued that students need a wide range of experiences with different types of problems, which should be reflected in the curricula choices made by teachers and prescribed by state/federal or local standards. According to these mathematicians and researchers, it is not merely enough to say that multiplication is repeated addition since that way of teaching simplifies mathematics and can cause misconceptions later in higher level mathematics classes. Rather, teaching should focus on conceptually understanding multiplication involves recursive thinking, or at the very least a rudimentary understanding of infinity.

Multiplication problems can be classified according to the nature of the quantities involved and the relation between them (Nesher & HersHKovitz, 1988). Greer (1997) lists four categories that primarily apply to problems involving the multiplication of whole numbers.

- equivalent groups (e.g., 2 tables, each with 4 children)
- multiplicative comparison (e.g., 3 times as many boys as girls)
- rectangular arrays (e.g., 3 rows of 4 children)
- Cartesian product (e.g., the number of possible boy-girl pairs)

Division conceptual understanding may be more complicated than multiplication. “To understand division requires more than knowledge of sharing out a collection equally; It requires an understanding of the relationship between the dividend, divisor, and the quotient, and the role of each in a division problem” (Correa, Nunes, & Bryant, 1998). Division is expressed mathematically in two distinctly different ways: partitive division, sometimes referred to as equal sharing or repeated division, means that you can solve a division problem by repeatedly subtracting the divisor from the dividend until you reach zero, counting how many times you subtracted to find the answer; while “partitioning by sharing out” refers to dividing a quantity into equal groups by distributing items one at a time, essentially “sharing” them out evenly among a given number of groups. If you are thinking about division this way, then $12 \div 3$ means 12 things shared into 3 groups and we wish to know how many things are there in each group. Measurement division, sometimes referred to as “chunking,” depicts division as a way to divide an amount into groups of a given size. If you are thinking about division this way, then $12 \div 3$ means 12 things divided evenly into groups of 3, and we wish to know how many groups we can make. Benson (2014) explains that young learners do not have adequate exposure to division concepts and resort to a calculator much too early. Benson advocates for teachers to use estimation, software, promise number line models of chunks, multiples of ten and encourage questions about strategies. Additionally, teachers should not shy away from encouraging a deep investigation of remainders early on in the teaching of division.

Mathematics education scholars call for more emphasis on conceptual understanding of division and multiplication. “Teaching activities for multiplication and division need to give young learners the opportunity to explore different representations of multiplications and division and to reason about connections between these” (Barmby, 2009, p. 60). Robinson and LeFevre (2012) have shown beneficial effects of practicing multiplication and factoring together for children in grades 5 and 6. Thus, fostering conceptual links between inverse operations should be incorporated into training. For multiplication, it seems intuitive to use repeated addition (Gray & Tall, 1994), and division as repeated subtraction, but that is only one of the 2 main structures for division - partitioning, sharing out or division by chunking (grouping). This can be challenging for students. Equivalent group division problems have classically been categorized as partition (sharing) and quotition (measurement) situations (Fischbein et al., 1985), yet teachers tend to focus on the former with little attention paid to the more abstract, and conceptually richer latter categorization of division. Teachers spend their time in grade 3 exploring partitive and quotative division word problems. However, teachers should focus on developing a relationship between multiplication and division rather than just explaining different types of division problems. (Downton, 2013) Unfortunately, multiplication and division are often taught separately and/or using one type of model. Downton (2013) and others we have mentioned above strongly assert the need to encourage a deeper connection between multiplication and division concepts and thinking by providing conceptually rich models and encouraging the connection between these concepts and models to learners.

The use of real life problems, using pictorial and concrete models have been advocated for decades in research (Davydov, 1995) There is also a growing body of literature regarding the benefits of specific models for teaching these topics. Many scholars point to the models and representations teachers use for multiplication and division. Scholars convey the importance of utilizing visual model representations such as arrays, set and area models, and number line models which helps students later make the important connection between multiplication and distributive property (Kinzer & Stanford, 2013). Hurst (2015) suggests that “the array is critically important in developing multiplicative thinking,” (p 11) while Cooper et al (2012) note various models include regions, lines, while discrete models include sets of objects that are beneficial in teaching multiplication and division concepts to young children.

In addition to textbooks addressed directly to students, there are numerous publications on the Polish publishing market addressed to teachers. Complementing the methodological guides dedicated to textbooks courses used by students, there are many popular publications that expand the range of methods for teaching children mathematics, including multiplication and division. Moreover, the scope of publications on mathematical education is particularly rich, which is associated with the stereotypical belief that students often experience learning difficulties in mathematics. Unfortunately, the quality of publications addressed to both teachers and parents is very diverse - from publications that propose learning multiplication tables through poems of questionable literary quality (e.g. Markowski, 2022) to

publications proposing creative methodological solutions. An increasingly popular method of learning multiplication tables is to work with factors and products presented not in columns (according to the principle one column - one factor multiplied by the others), but in tables. In this case, products of multiplication are often organized in four quarters (with maximum products - quarters I-25, quarters II and III - 50, quarters IV - 100). These structures determine the rhythm and stages of their learning by children. Another example of an educational material is a so-called board of one hundred numbers (presenting numbers on the board organized in a decimal system, i.e. In rows 1-10, 11-20, etc.), which is usually used to familiarize with multiples of numbers. Both of these mentioned aids are designed as a space for children's mathematical experiments involving the search for data, the study of mathematical properties (especially multiplication alternation), the development of the intuition of powers. There are also mathematical games, coloring according to instructions and finally - learning multiplication tables to aid the learning of multiplication and division. Despite many interesting methodological proposals related to the use of board multiplication tables, the practice of their use primarily concerns multiplication (without division). There are many models used to help learners understand multiplication algorithms, such as lattice, partial product, "helpful lines," "Grabowski's cards" lattice multiplication to name a few. These algorithmic methods are a broad and most intriguing area of research. However, they are beyond the scope of our study, since we are strictly interested in the ways in which multiplication and division are first introduced conceptually to learners, as this provides a bedrock for future learning.

Given the literature on teaching and learning the conceptual understanding of multiplication and division, we consider these concepts of mathematics important areas of inquiry. We wonder if textbook authors present multiplication and division in tandem at the beginning of a unit or present one at a time and only later make the connection? Second, what definition of each property is showcased prominently in the textbooks and how? For example, is multiplication shown as repeated addition? Is division only shown as partitive? We are curious to explore the models utilized in textbooks such as arrays, number lines, and/or real life story problems. We are also interested to see how multiplication and division are connected in textbooks in order to advance commutative and associative property understanding. To delve into the research to answer the above questions, we first need to understand how we might be able to do so using textbooks as a unit of analysis. In the following section we provide a summary of textbook analysis studies to ground our methodological choices.

Review of the research on Textbook Studies

Researchers have generally agreed that textbooks are a major conveyor of the curriculum and play a dominant role in modern education scenes across different school subjects (e.g. Baker, Knipe, Cummings, 2010; O'Keeffe, 2013; Yang & Ling, 2016). Moreover, in mathematics, Robitaille and Travers (1992) argued that a great dependence upon textbooks is "perhaps more characteristic of the teaching of mathematics than of any other subject" (p. 706). Mathematics textbooks serve as a key instrument for learning in a classroom and are highly relied upon, in varying degrees by international teachers. Mathematics textbooks could be one of the elements of these educational systems that may help to account for these results, given that textbooks have historically been considered and currently are considered a major element of mathematics education around the world (e.g. Haggarty & Pepin, 2002; Oates, 2014).

Indeed, in developing countries access to textbooks and mathematical achievement are strongly related (e.g. Fuller & Clarke, 1994; Lockheed et. al., 1986; Mullis et al. 2012). One of the first and more prominent studies in textbook comparisons came from Schmidt et al. (2007) volume, titled *A splintered vision: an investigation of U.S. science and mathematics education*. In this edited book, authors found that U.S. science and mathematics textbooks had many more topics, thereby devoting less time to each, than typical international textbooks. Baker et al. (2010) pointed out that mathematics textbooks can be regarded as the most accountable and important historical proof for the development of mathematics curriculum. In addition, research has shown that mathematics textbooks play a key role in the process of students' learning and teachers' teaching. As these studies and many more indicate, the quality of textbooks influences students' learning outcomes and mathematics achievement as well as teachers' teaching efficiency (e.g. Floden, 2002; Reys & Reys, 2010; Stein et al., 2007; Törnroos, 2005).

Textbooks provide a vision on what and how teachers should teach and how students will learn (Dole &

Shield, 2008; Fan & Zhu, 2007). Schmidt et al. (2007) define textbooks as micro-organizers of in-class activities, emphasizing their effects on teachers and teaching. Mathematics textbooks are viewed as powerful instructional tools that could act as sources for teaching activities and instructional ideas and hence, provide a particular version of curricular content in a specific sequence (Reys & Reys, 2010). Erbas et. al. (2012) explains that textbooks are widely accepted source of curriculum and provide three important roles for education:

- textbooks serve to guide the topics being taught
- textbooks help teachers organize topics and materials
- textbooks provide teachers with ideas and activities for how to teach their students.

Textbook-problem analysis has been conducted in several international studies thus far (e.g., Fuson et al., 1988; Mayer, Sims, & Tajika, 1995; Schmidt et al., 2007). Researchers have discovered that curriculum is one of the key factors that account for differences in student achievement. For example, Brown (2009) argued that textbooks affect teachers' utilization, instructional beliefs, and pedagogical understandings. Valverde et al. (2002) further explained that “[t]extbooks are designed to translate the abstractions of curriculum policy into operations that teachers and students can carry out. They are intended as mediators between the intentions of the designers of curriculum policy and the teachers that provide instruction in classrooms” (p. 2). Weinberg and Wiesner (2011) developed a reader oriented textbook theory that articulates how textbooks function in the classroom. For them, textbooks can be described in two ways: text oriented and reader-oriented. In the text-oriented view, readers take the knowledge presented in the text as objective and in the order in which it is given. These researchers analyzed textbooks based on visual design, content, and content presentation. They found that U.S. textbooks are strikingly different from Singapore in significant ways, such as organization of topics, explanations and methods given. Another example of differences found in international textbook studies comes from Voogt & Robin (2012) who found that mathematics textbooks in the United States frequently included images alongside mathematics concepts, yet lacked worked examples.

This growth of researchers interest in textbooks can be observed from the fact that the Third International Mathematics and Science Study (TIMSS) included an analysis of hundreds of textbooks and other curricular materials from about 50 countries, and it was believed to be the first time for a study of such a large scale to include textbooks as a major research subject (Schmidt et. al., 2007). Nevertheless, compared to other research areas in mathematics education, studies focusing on textbooks are still inadequate, and, with this concern, many researchers have called for more studies centering on textbooks (e.g., Fan et al., 2004a&b; Love & Pimm, 1996). Various studies have already been set up with a view to analyse math textbooks from two or more countries or regions and link the results of this comparative analysis to student learning outcomes (e.g. Depaepe et al., 2009; Mayer et al., 1995; Stigler et al., 1982). This study is part of this larger research effort that aims to investigate how, as the intended curriculum, mathematics textbooks in Mainland China, Singapore, and the U.S. represent problem solving for classroom teaching and learning. Our objective here is to examine how multiplication and division is introduced and thereby conceptually represented, in the following three countries that participated in TIMSS and PISA studies: Poland, Chinese and U.S. mathematics textbooks.

Methodology

The methodology used for this study is content analysis, which offers a systematic methodology to investigate the assumptions made within documents and affords a reflexive window into the cultural patterns, interests, and values a society holds. Content analysis technique that requires a systematic examination and interpretation to understand and make sense of data in print and digital materials (Bowen, 2009). As a methodology, content analysis can be classified as both quantitative and qualitative, since it incorporates numerical data analysis as well as an interpretative recursive approach to understanding the problematic. Krippendorff (2004) calls content analysis a “scientific tool” that “provides new insights for researchers to understand particular phenomena that informs practical actions” (p. 18). Content analysis has been greatly used in researching educational issues (e.g. Carnine & Jitendra, 1997; Taylor, 2009; Tenam-Zemach, 2010; Tasdemir, 2011) and to study educational policies and legislation is widespread internationally (e.g. Erdogan, Marcinkowski, & Ok, 2009; Eyler et al.,

2001). Specifically in mathematics education policy, there have been several researchers who employed content analysis for their research agendas. Of note, we have Higgins & Parsons (2009) study on New Zealand's numeracy policy implementation. Specifically in STEM policy, Lou et al. (2011) studied STEM knowledge of female Taiwanese high school students to explore the effects of problem-based learning strategies on the attitudes of female students towards STEM learning.

For this study, we gathered popularly used textbooks from three countries: Poland, United States, and Mainland China. We chose to limit our international analysis to these three countries due to distribution of international test scores, depicting the stark differences between these three countries. According to the 2022 PISA results, China scored 2nd overall (Singapore was first), Poland scored 15th and the United States scored 17 (which is markedly higher than in the recent TIMSS study, whereas the U.S. scored 37th).² According to the 2023 TIMSS report, United States children scored 517, China 607, and Poland (right in the middle) scored 546. To concentrate on the introduction of the concepts of multiplication and division, thereby capturing the conceptual understanding of these mathematical ideas as depicted in textbooks, we reviewed the national standards for each country.³

In the United States and China, these concepts are first introduced in third grade, but in Poland they are introduced in 2nd grade (the disparity is due to the start times of schooling, but children are the same age in these grades for all three countries). We then gathered our data points - textbooks - depending on their most popular use in each country. For the United States, we chose the three most used textbooks based on several reports:⁴ Everyday Mathematics, Eureka Mathematics, and IXL Learning Mathematics. While these three textbooks are different in organization and content, they all promote alignment to the U.S. national next generation mathematics standards. The U.S. Common Core standards, which are national and used in almost every state are comprehensive in their content standards for multiplication and division. There are seven standards devoted to this topic in 3rd grade within the Algebraic Operations and Thinking strand. To summarize, they explicitly ask teachers to make connections with multiplication and division and ask teachers to be sure to show the two ways to understand division. They also ask teachers to use story problems and more complex problem solving.⁵ During our analysis, we found that the U.S. textbooks are different from one another in significant ways, which we will discuss in the next section.

The Polish publishing market is varied (it's common for one publisher to introduce two or three textbooks' series to the market); therefore, it is difficult to find one unbiased ranking which would identify the most popular textbooks in Poland. For this study, we chose one textbook from the three biggest educational publishing houses in Poland. As the analysis shows, these textbooks are drastically different in content and organization. The Polish national standards for grades 1st to 3rd (children aged 7-10) do not state which grade particular factors and products should be learnt by students, therefore the process of learning multiplication and division may vary in classes using different textbooks - it often starts in 2nd grade and sometimes it spreads to 3rd grade. Again, Polish standards refer students only to "multiply and divide in memory using factors within the multiplication table; multiply factors smaller than 20 by 10 in memory"⁶ until the end of 3rd grade (age 9-10).

In China, which has a more uniform national curriculum, we only used one textbook. As with the other countries' textbooks, we collected the student textbooks (sometimes referred to as workbooks) rather than extensively look at the teacher edition because we are interested in the "intended" curriculum, i.e. what the learners will be interacting with. In the textbook, multiplication and division are introduced along with the "multiplication table". The Chinese multiplication table, also known as "九九乘法表" (jiǔ jiǔ chéng fǎ biǎo, "9-9 multiplication table"), is a 9x9 grid that displays the products of numbers 1 to 9 and is a foundational tool for learning multiplication in Chinese education. It is often required for mental

² <https://worldpopulationreview.com/country-rankings/pisa-scores-by-country>

³ <https://www.corestandards.org/Math/Content/3/OA/>

⁴ <https://edsources.org/2022/textbooks-do-we-know-which-ones-are-effective/678595>

⁵ <https://www.corestandards.org/Math/Content/3/OA/>

⁶ <https://isap.sejm.gov.pl/isap.nsf/DocDetails.xsp?id=wdu20170000356>

retrieval. The Multiplication Table is introduced and practiced along with the concept of multiplication in the order of “ $5 \rightarrow 2, 3, 4 \rightarrow 6 \rightarrow 7 \rightarrow 8 \rightarrow 9$ ”. Division is introduced in the order of “2, 3, 4, 5, $6 \rightarrow 7, 8, 9$ ”, with the last chapter introducing division with remainder. The chapter before “division with remainder” is about mixed operation (addition, subtraction, multiplication and division mixed operation, including operations with parentheses). There are 36 practices in total but they are not coded in this research. In the China textbook, each chapter is introduced following the structure below:

- a school-theme real life anchor problem (eg. total amount of students on a train/racing car in a park; distributing snacks to students in a field trip, etc)
- problem solving procedure to the anchor problem
- modeled practices to other real-life problems/story problems
- students’ independent practices
- chapter review and wrap up practices

We share this outline so that we can compare China’s textbook with Poland and the United States in the Findings section of this paper.

Table 1. Textbooks analyzed in the study

Country	Textbook	Publisher	# of pages devoted to multiplication and division / total pages in volume
China	Compulsory Education: Grade Two Math (Book 1 & 2)	People's Education Press (2017). ISBN: 978-7562165385	77/230
United States	Eureka Mathematics	Great Minds (2018) Grade 3. Module 1 (of 7), ISBN: 978-1640545939	99/150
	Everyday Mathematics	McGraw Hill (2015). Student Math Journal. Volume 1 & 2, Grade 3. ISBN: 978-0021430871	--/442 impossible to say since multiplication & division is scattered throughout
	IXL	IXL Learning (2023). The Ultimate Third Grade Math A Workbook. ISBN: 9781947569508	103/223
Poland	Me and my school (<i>Ja i moja szkoła</i>)	MAC. (2021) JUKA-91 Volume 1-7, Grade 2. ISBN: 978-83-8141-321-3	65/451
	Friends from school. Maths coursebook. (<i>Szkolni Przyjaciele</i>)	WSiP (2022). Volume 1 & 2, Grade 2. ISBN: 978-83-02-17409-4	34/190
	Explorers’ Elementary. New edition (<i>Nowy Elementarz Odkrywców</i>)	Nowa Era (2024). Volume 1 & 2, Grade 2. Warsaw. ISBN: 978-83-267-4518-8	29/156

Based on the literature review and research question, we developed a comprehensive code book. We separated the coding into three distinct categories to capture the nuances of the textbooks. First, we were interested to understand if the textbooks illustrated multiplication and division together, since the research indicates the importance of relaying the connection between these two processes/concepts. Second, we looked at the ways in which multiplication was illustrated, using the four categories created by Greer (1992). Last, we looked at the two distinct ways division is illustrated, as explained by Fischbein et al. (1985). Other codes emerged as needed if there were prevalent in the textbooks. Each code was given a clear definition and example. Test for reliability in our coding schema included all three researchers coded the first two chapters of one of the American textbooks. After some deliberation and fine tuning of our codebook, we were able to code the next chapter with a 99% accuracy. In two next sections, we illustrate our findings and discuss their implications. After carefully tallying each instance in which a code was shown, the tally conveyed below illustrated the total times a code occurred in a text. Problems in the textbook could, and often did, have multiple codes. For example, a story problem about multiplication could have a code of “story problem,” and “array model for multiplication,” if the story problem utilized an array visual to help the learner understand the concept.

Findings

The table below compares the textbooks from the three countries in our data set: China, Poland and the United States. The United States and Poland show an average for the three respective textbooks analyzed in these countries.

Table 2. Comparison of Multiplication and Division in Textbooks from China, the U.S., and Poland

Category	Code	China	US Avg	Poland Avg
Multiplication & Division Together	Instances shown together	19	6	18.7
	Inverse property shown	16	6	6
	Equation-based mental math	—	—	9.3
Properties of Multiplication	Commutative property	6	3.7	1.7
	Associative property	—	1	0.3
	Distributive property	—	—	3
Models	Repeated addition (multiplication)	25	7	7
	Equivalent groups (multiplication)	55	7.7	13.7
	Rectangular arrays (multiplication)	8	26.3	9
	Tape diagrams (multiplication)	14	3	4.3
	Arrays (division)	6	3	—
	Grouping models (division)	10	2.7	8.7
	Standard algorithm	13	4	—
Problem Types	Real-life problems	56	7.7	3
	Story/word problems	42	25.7	13
Mental Math	Multiplication	60	24	1
	Division	33	7.7	0.3
Division Types	Partition division	25	10	16
	Measurement division	26	11.7	9.7
	Division with remainder	several instances	—	—

When compared internationally, the Chinese textbook stands out for its structured, coherent, and conceptually progressive design. Unlike the fragmented or varied emphasis seen across U.S. and Polish textbooks, the single Chinese textbook offers a highly integrated approach, with 19 instances of multiplication and division shown together, 16 explicit references to the inverse relationship, and extensive use of story contexts (42) and real-life problems (56) to anchor conceptual learning to its highest potential. Visual modeling is consistent and intentional, including arrays, grouping models, and tape diagrams, often tied to mental strategies—e.g., 60 mental multiplication tasks and 33 for division. The Chinese textbook predominantly uses equivalent groups and repeated addition, while the U.S. and Polish textbooks use a wide variety of visual models. Another important difference is the emphasis on mental math in the Chinese textbook in multiplication (60 instances) and division (33 instances) problems in comparison to the United States (24, 7.7, respectively) and Poland (1 and .3 respectively).

While the U.S. textbooks are split in focus—*Eureka* being model-heavy, *IXL* skill-based, and *Everyday Mathematics* context-rich—and the Polish textbooks demonstrate a strong focus on mental math and integration of operations. In contrast, the Chinese textbook is unique in its balance of procedural fluency, conceptual depth, and real-world application. Additionally, China's inclusion of a division-with-remainder unit, absent in the other nations' texts, suggests a more advanced treatment of foundational operations at an earlier stage. These patterns reflect broader curricular priorities: coherence and rigor in China, variation and specialization in the U.S., and blended integration in Poland.

Table 3. Detailed Comparison of U.S. Textbooks

Category	Code	Eureka	IXL	Everyday Math
Multiplication & Division Together	Shown together / Inverse property	0	6	12
Properties of Multiplication	Commutative property	8	0	3
	Associative property	0	3	0
Problem Types	Real-life problems	6	3	14
	Story/word problems	36	15	26
Mental Math	Multiplication	6	28	38
	Division	0	11	4
Models	Repeated addition (multiplication)	18	2	1
	Equivalent groups (multiplication)	12	5	6
	Rectangular arrays (multiplication)	43	10	26
	Tape diagrams (multiplication)	8	0	1
	Arrays (division)	7	2	0
	Grouping model (division)	6	2	0
	Standard algorithm (multiplication)	0	4	0
	Standard algorithm (division)	0	8	0
Division Types	Partition division	16	8	6
	Measurement division	23	2	10
	Shown together (division)	5	0	0

The comparative analysis of the three U.S. mathematics textbooks—Eureka Math, IXL Learning, and Everyday Mathematics reveals notable variation in how multiplication and division are introduced and emphasized. While *Everyday Mathematics* shows the greatest attention to presenting multiplication and division together (12 instances) and includes the most real-life and story/word problems (14 and 26 respectively), *Eureka Math* provides significantly more opportunities for conceptual modeling, particularly through rectangular arrays (43 instances) and repeated addition (18). In contrast, *IXL Learning* emphasizes procedural fluency and mental math strategies, with 28 instances of mental multiplication and 11 for division, suggesting a strong focus on computation rather than representation. Interestingly, the standard algorithms for multiplication and division appear only in *IXL*, indicating a unique focus on formal procedures. While all three textbooks include elements of partition and measurement division, *Eureka Math* stands out with the highest number of modeled strategies for division (e.g., grouping, tape diagrams, and arrays). These findings suggest that *Eureka* emphasizes conceptual depth, *Everyday Mathematics* leans on contextual richness, and *IXL* prioritizes computational skill, pointing to diverging instructional philosophies across commonly used U.S. curricula.

Table 4. Detailed Comparison of Polish Textbooks

Category	Code	MAC	WSiP	Nowa Era
Multiplication & Division Together	Shown together	44	9	3
	Inverse property	8	5	0
	Equation-based mental math	19	7	2
Properties of Multiplication	Commutative property	1	0	4
	Associative property	0	1	0
	Distributive property	6	3	0
Problem Types	Real-life problems	2	6	1
	Story/word problems	24	5	2
Mental Math	Multiplication	0	1	0
	Division	0	0	1
Models	Repeated addition (multiplication)	12	6	3
	Equivalent groups (multiplication)	6	18	17
	Rectangular arrays (multiplication)	8	15	4
	Tape diagrams (multiplication)	1	11	1
	Grouping model (division)	5	7	14
	Division equation (mental math)	11	4	8
Division Types	Partition division	22	7	19
	Measurement division	10	0	0

The analysis of the three Polish textbooks—MAC, WSiP, and Nowa Era—demonstrates a stronger integration of multiplication and division, with MAC especially emphasizing their connection (44 instances compared to 3 in *Nowa Era*). As mentioned, equation-based mental math is quite a common task for students, especially in MAC textbooks (19 instances). Story/ word problems are even more popular in MAC textbooks with 24 instances (compared to 5 in WSiP and 2 in *Nowa Era*). Unfortunately, the real-life problems tasks are significantly less popular (6 instances in WSiP, 2 in MAC, 1 in *Nowa Era*). The analysis of textbooks shows three patterns of introduction multiplication - repeated addition

(12, MAC), comparison of addition and multiplication as two procedures of counting (12 instances in MAC) as well as equivalent groups (18, WSiP; 17, Nowa Era). Less popular, yet still very common are visual models of multiplication - rectangular arrays (15, WSiP) and tape diagrams (11, WSiP). Properties of multiplication are not a popular topic in Polish textbooks with 4 instances of commutative property in Nowa Era (1 in MAC, 0 in WSiP), 1 of associative property in WSiP (0 in MAC and Nowa Era) and 6 of distributive property in MAC (3 in WSiP, 0 in Nowa Era). Similar to multiplication, division is introduced with one of two models: partition division (22 instances in MAC; 19 in Nowa Era; 7 in WSiP) which is more popular teaching approach in Poland and measurement division - less common, yet this model was used as a first one in MAC textbooks. Division is also explained on the grouping model, often visualized with illustrations (14, Nowa Era). The order of introduced factors differs in each textbook. In WSiP multiplication is introduced on the examples of mixed factors, only later the rhythm occurs and cover multiplication and division by 4, 5, and 7 with maximum product of 50, followed by factors 6, 7, 8 and 9 with products bigger than 50. The similar model of explaining multiplication/division with mixed factors occurs in Nowa Era, however this is the only textbook which introduces operations like 30:1, 30:30, and 100 as a factor with maximum product of 1000. MAC textbooks seem to order factors quite clearly starting with multiplication by 2 ($n \times 2$ when $n < 6$, and $n \times 2$ when $n > 6$), then the pattern repeats for $n < 6$ with factors 3, 4 and 5.

Discussion

This study indicates the potential differences in the way in which multiplication and division are conceptualized in three countries. While there are differences in each of the three textbooks analyzed in the United States and in Poland, there are some general differences between them and the Chinese textbook. The most glaring contrast is the overall presentation of the content. The Chinese student textbook offers a very clear and organized approach to learning multiplication and division with each section following similar pacing and offering concrete real-life problems. In contrast, the Polish as well as the U.S. student textbooks varied (in each textbook itself) how the content was presented and offered little real-world applications.

Perhaps less is more we wonder as researchers. The U.S. and Polish texts, trying to offer a more interdisciplinary and/or inter-related content, may confuse children inadvertently. There were some unique features in the Chinese textbook we would like to highlight. First, there was a song for learning the multiplication facts fluently, which was presented at the beginning of the lessons. The song was easy and fun to learn since Chinese words for numbers are much easier than American or Polish and follow a rhythm-like pattern. Each section in the Chinese textbook began with a story, a real-life story about school or children's family life. When division was introduced, the vocabulary was emphasized first and the remainder was discussed right away, related again to real life stories. This was very much different from either the U.S. or Polish text, where there were no problems in division with remainders.

In Poland, the textbooks series differed in the structure of maths' programme. Nowa Era and WSiP were only maths related books, but MAC mixes all educational areas in one integrated course. It affects story/word problems as they don't reflect real life maths' problems but relate more on the general topic of the week (e.g. students are to divide the medieval warriors in the equal groups in a task related to Polish history topic or multiply the amount of blood ordered by types in bags). As story/word problems are the most common exercise in MAC textbooks, their educational potential is often misused as they are just another abstract assignment rather than an interesting challenge. On the other hand, students are often asked to identify the maths problem by stating the question to the unfinished story/word problem which seems an interesting task. In all textbook students learn math terms related to multiplication and division (e.g. factor, product, dividend, divisor, quotient) and the visual model of multiplication and division is very popular. Illustrations supporting the process of understanding multiplication and division are especially popular in Nowa Era textbooks, but they are also often used in other series. Another example of visual models are arrow graphs, which illustrate the relation between multiplication and division and "tree models" which help students to learn order of operations. Moreover, MAC and WSiP introduce table of factors and products which put numbers in order and help students to analyze the commutative property of multiplication. Another interesting finding refers to a way of introducing division types - as it is very common in Poland to start with partition (as Nowa Era and WSiP do), in MAC textbooks measurement division is the first model explained. All of the textbooks introduce the order of operations,

yet it seems that MAC the rules are explained in the most detailed (starting with multiplication and addition, then multiplication and subtraction, division and addition, division and subtraction, finishing with all operations mixed together).

In contrast, the U.S. textbooks seemed to, to use a metaphor, “throw everything but the kitchen sink” at the children, hoping that one of the methods would be relatable. While many models were represented for multiplication and division, they were scattered and not uniformly presented so that learners could view each at time to develop meaningful understanding before being introduced to yet another model. Additionally, story problems were scattered throughout the textbook with very little presentation of real-life problems as the Chinese textbooks do. As in the Polish textbooks, no mention of remainders is discussed in any of the 3 textbooks analyzed. A glaring contrast between the U.S. textbooks and Polish and Chinese was the lack of depiction of multiplication and division shown conceptually together. This signifies a gap in conceptual understanding that U.S. learners may have since they learn multiplication and division primarily as individual mathematical concepts and thereby do not understand the importance of the inverse property. This also shows a lack of true alignment with the Common Core standards. Another difference we would like to emphasize between the U.S. textbooks and Poland and China is the use of multiplication models. While the Chinese textbook emphasizes repeated addition and equivalent groups, the U.S., on average, textbooks emphasize rectangular arrays. Although models are used frequently in the U.S. textbooks, the frequency of rectangular arrays may indicate a lack of conceptual modeling that can aid learners in understanding multiplication fluently.

Conclusion

This study illuminates the differences between how three countries present multiplication and division to children for the first time in formal mathematics education. While some of the findings may appear intuitive or obvious, several important discoveries have been discussed in the above two sections of this paper. The presentation of multiplication and division for young children relates to conceptual understanding of these important mathematical concepts and paves the way for proficiency in mathematics at higher levels.

Certainly, there are several limitations to this study that are noteworthy. First, the sample size we used is not representative of the diverse countries that participated in the TIMSS and PISA international tests. Originally, we chose the three countries because we felt they were relatively distributed along test scores and therefore may show different perspectives in curricula choices. However, limiting to only three countries may leave more questions than answers about the way in which intended curricula such as textbooks may influence a country’s overall mathematics education testing results. Further, our analysis of three textbooks in the United States and three in Poland may not reflect the nuances in textbooks used throughout these countries. While we did choose the most used curricula materials, a more accurate picture could be achieved by reviewing more textbooks from different publishers. Perhaps understanding how each state or region utilizes textbook materials differently could illuminate more differences than we have uncovered thus far in our study. Our second constraint was the material itself. Since we choose to utilize the intended curriculum, we limited ourselves to the workbooks students would be utilizing in each country. While we did have access to the teacher edition of these workbooks and did review them to get a sense of the students’ experiences, we did not code the teacher edition. This was a methodological choice given the research question. Our objective was to explore the way in which students interacted with the concept of multiplication and division as it is introduced in schools and worked with at home and therefore, coding the problem types and distribution was the best way to code for such phenomena. The detailed code book we created for this study could offer further information on other educational phenomena such as classroom lessons, but that would involve a more ethnographic study. TIMSS does offer video analysis potential, but those are limited to 6th and 8th grade, and not the 2/3rd grade we were interested in here. Even though the study has the above limitations, the potential for utilizing the methodology we have created is substantial.

This small study does illuminate the differences between the intended curriculum is comparatively in various countries and even within one country. We hope this study encourages other researchers to analyze textbooks further, perhaps focusing on other important concepts in childhood mathematics education, such as fractions and geometry, and measurement. These types of studies can greatly aid a

country, like the United States, reform its educational policies and standards. It can also help textbook publishers and school stakeholders in choosing material that can improve mathematical proficiency for all students.

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