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*Original Paper*

# Stochastic Valuation of a Company

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**Abstract**

This paper tries to give a summarized vision of a potential applicable model for the valuation of a company within the context of a possible bankruptcy in the horizon of 10 years, in an environment of buying and selling a business.

**Keywords:** Company valuation, Montecarlo Simulation, Stochastic Optimization, Probabilities of bankruptcy, survival, Discounted Cash Flow.

## 1. Objective

Development of a company valuation model under three different scenarios, which contemplates the probable bankruptcy of the company given the economic environment in which it operates.

The scenarios are:

a. Deterministic valuation using static failure/break probabilities.

b. Stochastic valuation using an exponential failure probability generator under two different Monte Carlo simulation hypotheses for the parameter:

   i. Uniform distribution.

   ii. Triangular distribution.

   iii. Valuation under stochastic optimization maximizing and minimizing the expected value of risk-adjusted cash flows, by introducing two decision variables, the interest rate (i) to discount and the growth rate (g).

## 2. Background

There is no doubt that any company can go bankrupt at any time, given an adverse economic condition. When valuing a company, for example, within a 10-year horizon, if the potential risk of bankruptcy is not taken into account, it tends to be overestimated. Generally, the value of the company adjusted for a probability of failure, makes the value of the company much lower, obviously depending on the probability distribution used. In that sense, this work is a fairly simple theoretical contribution on how to stochastically value a company in order to determine a fair value, within the context of a purchase-sale negotiation.

## 3. Definition of the Mathematical Model

To determine that the theoretical value of a company under the potential risk of bankruptcy, within a given horizon, is lower than regular valuation, the following variables are defined:

a. Time horizon: 10 years

b. Growth rate (g):10%

c. Free Cash Flow (FC(t))

d. Expected value of free FC E(F((t))

e. Present Value of Free Cash Flow FC(t): PV(F(t))

f. Present Value of Expected Value of Free Cash Flow: PV(E(FC(t))

## 4. Assumptions and Hypotheses

It is assumed that the company's bankruptcy/failure rates exhibit an exponential distribution in the 10-year time domain. It is assumed that these quasi-probability rates come from a pilot study that supports the idea of an exponential decay based on the fact that the company intrinsically has a greater probability of failure in the first years of life and as time goes by and experience is gained. Of market and positioning, the success rate tends to rise until it stabilizes at some point in the future, which translates into a very low probability of failure in recent years, before reaching a potential decline in the vegetative life cycle of companies.

### Basic Equations of the Model

a. Generating function of the exponential failure rates: $f(t \mid \lambda) = \propto e^{-\lambda t}$

b. Survival odds given failure: $Prob(S_t \mid f_t)$

c. Probabilities of failure conditional on survival until$(t)$: $Prob(f_t \mid S_{t-1})$

d. Free Cash Flow Growth Rate: $g$

e. Free Cash Flow (Note 1): $FC(t)$

f. Expected value of CF: $E(FC(t))$

g. Present value of free cash flow: $VP\big(FC(t)\big)$

h. Present Value of Expected Free Cash Flow: $VP(E(F(t))$

i. Discount rate to update the $FC(t)$: i

j. $f_t$, $S_t$: Failure and survival event in t.

$$FC_t = FC_0(1+g)^t = FC_{t-1}(1+g) \quad (1)$$

$$VP(FC) = \sum \frac{FC_t}{(i+t)^t} \quad (2)$$

$$E(FC) = \sum FC(t)P_f(t) \quad (3)$$

$$VPE\big(FC_{(t)}\big) = \sum \frac{FC_{(t)}P_f(t)}{(1+i)^t} \quad (4)$$

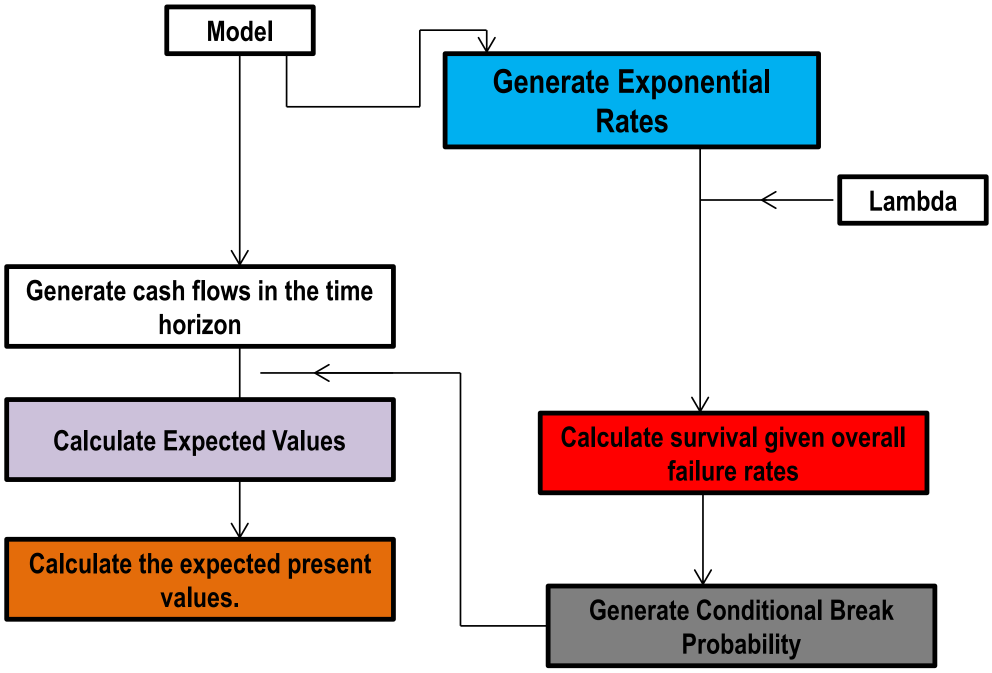
$$Prob(S_t) = 1 - Prob(f_t) \quad (5)$$

$$Prob(S_t) = \prod \big(1 - Prob(f_t)\big) \quad (6)$$

$$Prob(S_t) = \prod (1 - Prob(f_t)) \quad (7)$$

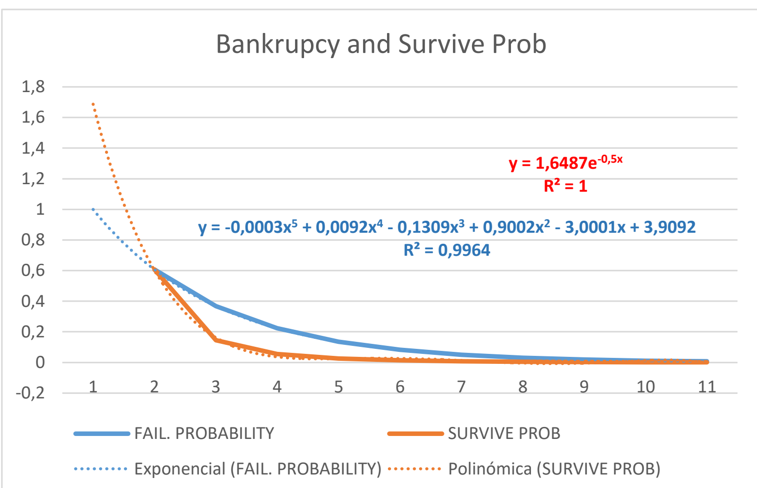
$$Prob\,(f_t \mid S_{t-1}) = Prob(S_{t-1})Prob(f_t) \quad (8)$$

### Monte Carlo Simulation Process for the Previous Model




**Deterministic Scenario Illustrative Example/Excel**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| LAMBDA | 0,5 | | g 0,1 | | | i | 0,15 |
| | | | 100000 | | | | |
| | 0 | | | | | | |
| TIME | FAIL. PROBABILITY | SURVIVE PROB | FC(T) | E(FC(T)) | VP(FC(T)) | VP(E(FC(T))) | |
| 1 | 0,60653066 | 0,60653066 | 110.000,00 | 66.718,37 | 95.652,17 | 58.015,98 | |
| 2 | 0,367879441 | 0,144749281 | 121.000,00 | 17.514,66 | 91.493,38 | 13.243,60 | |
| 3 | 0,22313016 | 0,055496947 | 133.100,00 | 7.386,64 | 87.515,41 | 4.856,84 | |
| 4 | 0,135335283 | 0,026149905 | 146.410,00 | 3.828,61 | 83.710,39 | 2.189,02 | |
| 5 | 0,082084999 | 0,013714204 | 161.051,00 | 2.208,69 | 80.070,81 | 1.098,11 | |
| 6 | 0,049787068 | 0,007635295 | 177.156,10 | 1.352,64 | 76.589,47 | 584,78 | |
| 7 | 0,030197383 | 0,004400475 | 194.871,71 | 857,53 | 73.259,49 | 322,38 | |
| 8 | 0,018315639 | 0,002588425 | 214.358,88 | 554,85 | 70.074,30 | 181,38 | |
| 9 | 0,011108997 | 0,001541205 | 235.794,77 | 363,41 | 67.027,59 | 103,30 | |
| 10 | 0,006737947 | 0,000924403 | 259.374,25 | 239,77 | 64.113,35 | 59,27 | |
| SUM | | 0,863730799 | **1.753.116,71** | **101.025,17** | **789.506,37** | **80.654,65** | |
| | | | | | **MULTIPLERATIO** | **10,22%** | |




Markovian Probabilities tree

| | | |
|---|---|---|
| | | 0,055496947 |
| | 0,144749281 | 0,944503053 |
| 0,606531 | 0,855250719 | |
| 0,393469 | | |

### 5.a Simulation/Deterministic Results

a. With a parameter of λ=50%, to generate the failure rates, the following results are observed, the probabilities vary from 60.65% in year 1 to 0.67% in the last year of evaluation (within the horizon established in the modelling).

b. The probability of failure $t$ for all $t \epsilon \{1,2, \dots 10\}$ conditional on survival $(t-1)$ varies from 60.65% to 0.092%.

c. With a growth rate of the flows in the year-on-year by the of order $g = 10\%$, in the last year 10, **259,374.25** um were reached , starting from a $FC(0) = \mathbf{100.000}$ um

d. The expected value of the cash flow is 101,025.17 or 5.76% (of the total cash flow)

In nominal terms not adjusted for interest, it is concluded that the expected total cash flow is almost 6% of the total cash flow in the same period.

In terms of the present value of the expected vs. unexpected total cash flows, the following is obtained for this scenario:

| $\sum_{\forall t} VP(FC(t))$ | $\sum_{\forall t} E(VP(F(t)))$ |
|---|---|
| 789,506.37 | 80,654.61 |

The present value of the company using a discounted cash flow without terminal value would be in the order of 789.5 um, while the adjusted value is in the order of 10.21% of the previous regular value, that is, 80.65 um.

### 5.b Stochastic Simulations

All the previous simulations were deterministic, although static probabilities were used from the dynamic perspective. Now we model under Monte Carlo simulation the parameter λ that generates the exponential probabilities.

If we consider the previous scenario as the base scenario and model a lambda of the function that generates exponential rates, with a triangular distribution, we would obtain the following results for the multiple and the PV(FC), on the regular valuation value and its variability.

With $\lambda =$ { 0.1 Minimum; 0.5 Most likely; 0.7 Maximum }

66,945.94

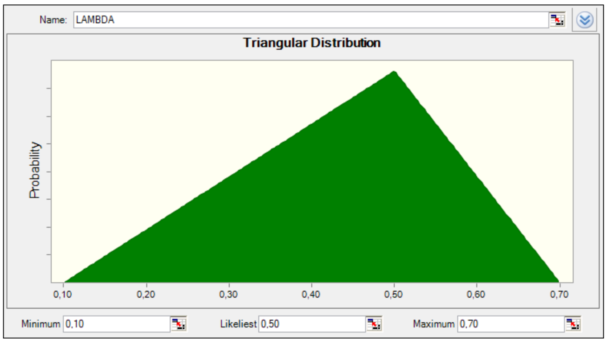
| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| LAMBDA | 0,5 | | TASA BENEFICIO | 0,13 | | TASA INTERES | | 0,17 |
| | | | | 100000 | | | | |
| | 0 | | | | | | | |
| TIEMPO | PROB.FALLA | SUPERV | | FC(T) | E(FC(T)) | VP(FC(T)) | VP(E(FC(T))) | |
| 1 | 0,60653066 | 0,606531 | | 113.000,00 | 68.537,96 | 96581,20 | 58.579,46 | |
| 2 | 0,367879441 | 0,144749 | | 127.690,00 | 18.483,04 | 93279,28 | 13.502,11 | |
| 3 | 0,22313016 | 0,055497 | | 144.289,70 | 8.007,64 | 90090,24 | 4.999,73 | |
| 4 | 0,135335283 | 0,02615 | | 163.047,36 | 4.263,67 | 87010,23 | 2.275,31 | |
| 5 | 0,082084999 | 0,013714 | | 184.243,52 | 2.526,75 | 84035,52 | 1.152,48 | |
| 6 | 0,049787068 | 0,007635 | | 208.195,18 | 1.589,63 | 81162,51 | 619,70 | |
| 7 | 0,030197383 | 0,0044 | | 235.260,55 | 1.035,26 | 78387,73 | 344,94 | |
| 8 | 0,018315639 | 0,002588 | | 265.844,42 | 688,12 | 75707,80 | 195,96 | |
| 9 | 0,011108997 | 0,001541 | | 300.404,19 | 462,98 | 73119,50 | 112,69 | |
| 10 | 0,006737947 | 0,000924 | | 339.456,74 | 313,79 | 70619,69 | 65,28 | |
| | SUMA | 0,863731 | | 2.081.431,65 | 105.908,85 | 829993,71 | 81.847,67 | |
| | | | | | | TASA | 9,86% | |

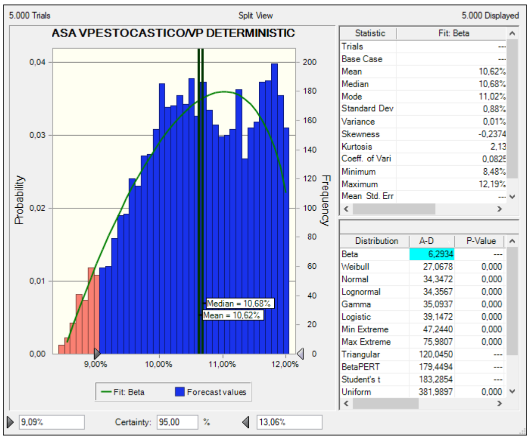
ASSUMPTIONS

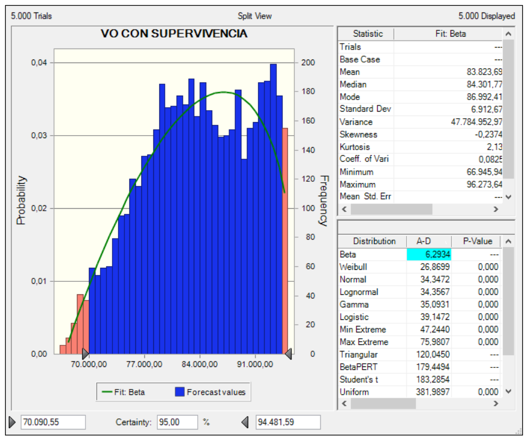
DECISION VARIABLE

RESULTS

**Results of the multiples of the values of the company expected vs. regular** $\left(\frac{VPE}{VP}\right)$




a. Under this scenario, we conclude that the multiple of the simulated cash flows in terms of the expected value with respect to the present value of regularly calculated flows varies between 8.48% and 12.19%, with a mean of 10.62%.

b. The company's expected present value is in the order of 83.8 um .

c. The resulting distributions adjusted for both the multiple of the value of the company and for the value adjusted by the probability of failure would be a beta distribution with an AD fit test of 6.29 for both multiples and value.

The foregoing gives an interesting margin of negotiation of the potential value of the company, obviously conditioned to the parameter generated from the failure rates $\lambda$, in the assumption or hypothesis in which it makes sense to assume, based on previous experience, a triangular distribution. The foregoing undoubtedly defines a possible negotiation margin of a company valuation adjusted for bankruptcy risk.

Another possible scenario that is generally used when there is no more information on the variability of the parameter value is to assume a uniform distribution, such as:

UNIFORM $$\lambda = \begin{cases} 0{,}1\ MÍNIMO \\ 0{,}7\ MÁXIMO \end{cases}$$

Under this scenario, when running the simulation we obtain the following:

1. The multiple rate varies at 95% statistical confidence between 8.64% and 13.06%.
2. The adjusted enterprise value in this case ranges from 67,059.61 to 9,506.92 um.
3. It is interesting to know that although the distributions resulting from the multiple and the expected value of the company continue to be a beta, the geometry of the distribution is obviously different.

**5.c Results under Stochastic Optimization**

The two previous scenarios obviously demonstrate with relative scientific objectivity that the expected value of the company given the vector of bankruptcy probabilities reduces the deterministic present value of the company to almost 10% and in that sense it is a statistical confirmation of the soundness of the multiple subject to the hypotheses and assumptions used.

In addition to the 2 scenarios already seen above, a new one could be created by introducing a couple of **decision variables to the model** that allows MINIMIZING/MAXIMIZING the expected present value of the random cash flows generated by the Montecarlo simulation.

In that sense, depending on the position adopted by the researcher to MINIMIZE the expected present value for the buyer and MAXIMIZE for the seller, the stochastic objective function would be:

**Minimization of the expected value of the Function (EVP) Buyer Vision:**

min $\boldsymbol{E}\{VPE(FC_t)\}$

$arg: (i, g)$

**Maximizing the Expected Value of the Function (EVP) Vendor Vision:**

$min \quad \boldsymbol{E}\{VPE(FC_{(t)})\}$

$arg: (i, g)$

$(i, g)$:Within the field of variation or domain of the plane$(ixg)$

When the interest rates and the growth rate are varied respectively in the domain $i\epsilon[12\%, 17\%] g\epsilon[7\%, 13\%]$,, the following matrices are obtained:

**Solutions for maximization/ minimization**

The vector of the first 15 feasible solutions modeling 5000 samples under Monte Carlo, for each distribution associated with the maximization or minimization of the total expected value converge in the following solution:

| MAX | MIN |
|---|---|
| 91.406,11 | 79.677,65 |

(Note 3)

Additionally, the dimension maximization and minimization matrices (5x7) are shown below showing the **respective sensitivities of the potential value of the company** with respect to the variations of the interest of the growth rate g.

**Maximization Matrix for the Expected Value of the Company**

**(g)**

**(i)**

| | | | | | | |
|---|---|---|---|---|---|---|
| 84.589,14 | 85.701,40 | 86.824,79 | 87.959,59 | 89.106,08 | 90.264,58 | 91.435,40 |
| 83.545,94 | 84.638,12 | 85.740,98 | 86.854,77 | 87.979,78 | 89.116,28 | 90.264,58 |
| 82.530,29 | 83.603,16 | 84.686,27 | 85.779,87 | 86.884,23 | 87.999,62 | 89.126,30 |
| 81.541,04 | 82.595,31 | 83.659,41 | 84.733,60 | 85.818,10 | 86.913,19 | 88.019,11 |
| 80.577,10 | 81.613,44 | 82.659,24 | 83.714,73 | 84.780,13 | 85.855,69 | 86.941,65 |
| 79.637,43 | 80.656,49 | 81.684,64 | 82.722,11 | 83.769,12 | 84.825,88 | 85.892,64 |

**Minimization Matrix for the Expected Value of the Firm**

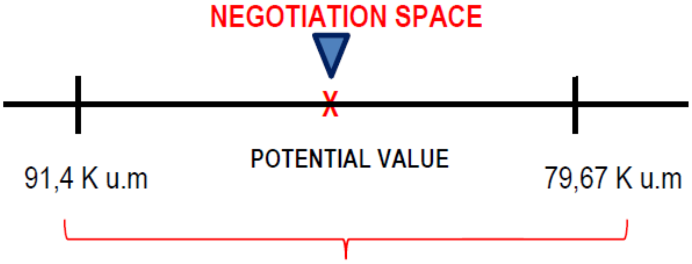
**(g)**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | 84.424,73 | 85.536,89 | 86.660,23 | 87.795,04 | 88.941,61 | 90.100,25 | 91.271,26 |
| **(i)** | 83.381,68 | 84.473,71 | 85.576,46 | 86.690,21 | 87.815,23 | 88.951,81 | 90.100,25 |
| | 82.366,22 | 83.438,89 | 84.521,85 | 85.615,36 | 86.719,67 | 87.835,07 | 88.961,83 |
| | 81.377,21 | 82.431,23 | 83.495,13 | 84.569,17 | 85.653,58 | 86.748,63 | 87.854,57 |
| | 80.413,54 | 81.449,59 | 82.495,15 | 83.550,44 | 84.615,70 | 85.691,16 | 86.777,09 |
| | 79.474,17 | 80.492,90 | 81.520,77 | 82.558,01 | 83.604,82 | 84.661,45 | 85.728,12 |

## 5. Conclusions

Without wishing to pretend to have covered all the alternatives that may exist in this important issue in the financial and economic area and being aware that this model, like any other, only reflects a vision of so many others that may coexist, both in its design and in its complexity , the matrices derived from the stochastic simulation for the two decision variables, income growth rate and interest rate to be discounted, can be taken into account in a discussion table even as a business game between buyer and seller to reach to more values of indifference interesting for both and from there proceed to a solid negotiation convenient for the parties, obviously conditioned to some basic assumptions previously established as in this case, in which **a uniform distribution was used** for lambda, generator of the probabilities of bankruptcy and two perfectly known and jointly defined decision variables with 2 opposite optimization criteria in a statistical result of the simulation such as the total present value of the expected value of the free cash flows.

The starting value of the seller as we saw in the maximization scenario is 91,406.11 and for the buyer in the minimization scenario it was 79,677.65. It is very possible that an intermediate value between the two will end up being negotiated, probably within the range described above in the previous matrices.



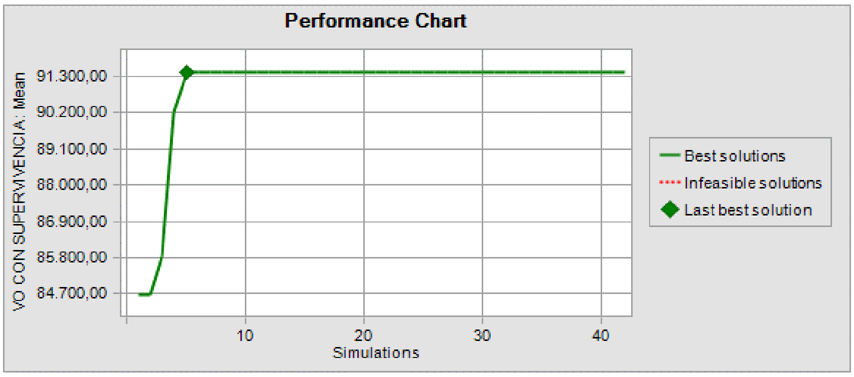

The value of the final company traded in theory would be found in the calculated sensitivity matrices. A curve fit to the matrix would even allow interpolations to determine the price of different values of i and g.

Probably a potential Value of 85.7 K um, is indifferent for both players and the negotiation is closed.

Annex I

**A. CONVERGENCE VECTORS OF FEASIBLE SOLUTIONS FOR MAXIMIZATION ( i,g )**

| | | | Objective | Decisión Variables | |
|---|---|---|---|---|---|
| | | | Maximize Mean | i | g |
| Rank | Solution # | | VO CON SUPERVIVENCIA | E1 | 1 |
| 1 | 5 | | 91.406,11 | 0,13 | 0,12 |
| 2 | 4 | | 90.233,04 | 0,12 | 0,12 |
| 3 | 15 | † | 89.509,41 | 0,13 | 0,13 |
| 4 | 7 | † | 87.722,01 | 0,13 | 0,14 |
| 5 | 17 | † | 87.712,09 | 0,12 | 0,13 |
| 6 | 31 | † | 87.702,00 | 0,11 | 0,12 |
| 7 | 27 | † | 86.626,06 | 0,13 | 0,15 |
| 8 | 16 | † | 86.606,76 | 0,12 | 0,14 |
| 9 | 28 | † | 86.587,13 | 0,11 | 0,13 |
| 10 | 22 | † | 86.567,14 | 0,10 | 0,12 |
| 11 | 3 | | 85.853,38 | 0,13 | 0,17 |
| 12 | 32 | † | 85.559,63 | 0,13 | 0,16 |
| 13 | 12 | † | 85.531,46 | 0,12 | 0,15 |
| 14 | 37 | † | 85.502,80 | 0,11 | 0,14 |
| 15 | 9 | † | 85.473,64 | 0,10 | 0,13 |



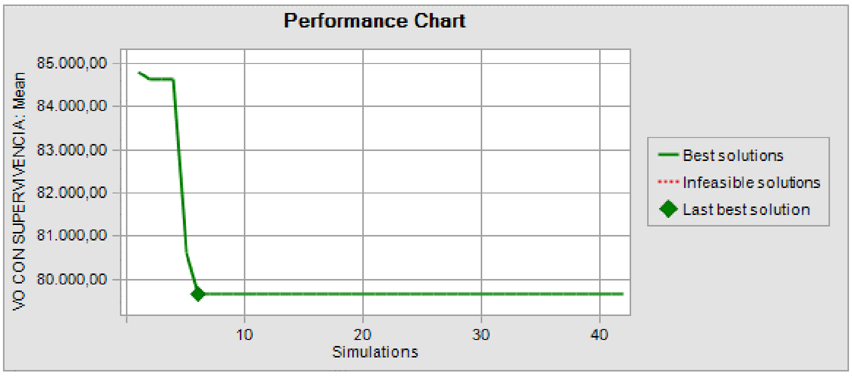

**Summary:**

After 42 solutions were evaluated in 59 seconds, the Mean of VO CON SUPERVIVENCIA was improved from 84.692,48 to 91.406,11, a change of 7,93%

Annex II

**B. CONVERGENCE VECTORS OF FEASIBLE SOLUTIONS FOR MINIMIZATION ( i,g )**

| | | Objective | Decisión Variable | |
|---|---|---|---|---|
| | | Minimize Mean | i | g |
| Rank | Solution # | VO CON SUPERVIVENCIA | E1 | 1 |
| 1 | 6 | 79.677,65 | 0,07 | 0,17 |
| 2 | 5 | 80.618,38 | 0,07 | 0,16 |
| 3 | 14 † | 81.543,08 | 0,08 | 0,17 |
| 4 | 26 † | 82.730,38 | 0,07 | 0,15 |
| 5 | 15 † | 82.802,60 | 0,08 | 0,16 |
| 6 | 19 † | 82.873,62 | 0,09 | 0,17 |
| 7 | 10 † | 83.716,96 | 0,07 | 0,14 |
| 8 | 22 † | 83.781,78 | 0,08 | 0,15 |
| 9 | 24 † | 83.845,52 | 0,09 | 0,16 |
| 10 | 9 † | 83.908,20 | 0,10 | 0,17 |
| 11 | 2 | 84.635,16 | 0,07 | 0,12 |
| 12 | 28 † | 84.729,35 | 0,07 | 0,13 |
| 13 | 1 | 84.779,80 | 0,10 | 0,15 |
| 14 | 38 † | 84.786,38 | 0,08 | 0,14 |
| 15 | 27 † | 84.842,44 | 0,09 | 0,15 |




Annex III

| | |
|---|---|
| **Run preferences:** | |
| Stochastic optimization (with simulation) | |
| Low-confidence testing on | |
| Maximum trials per simulation | 5.000 |
| Monte Carlo | |
| Random seed | |
| Precision control on | |
| Confidence level | 95,00% |

| **Run statistics:** | |
|---|---|
| Total optimization time (min:sec) | 00:59 |
| Number of simulations | 42 |
| Stopped by | |
| Trials limit reached | 5 |
| Precision control | 0 |
| Low-confidence testing | 37 |
| Infeasible constraints | 0 |
| Seconds/simulation (average) | 1 |

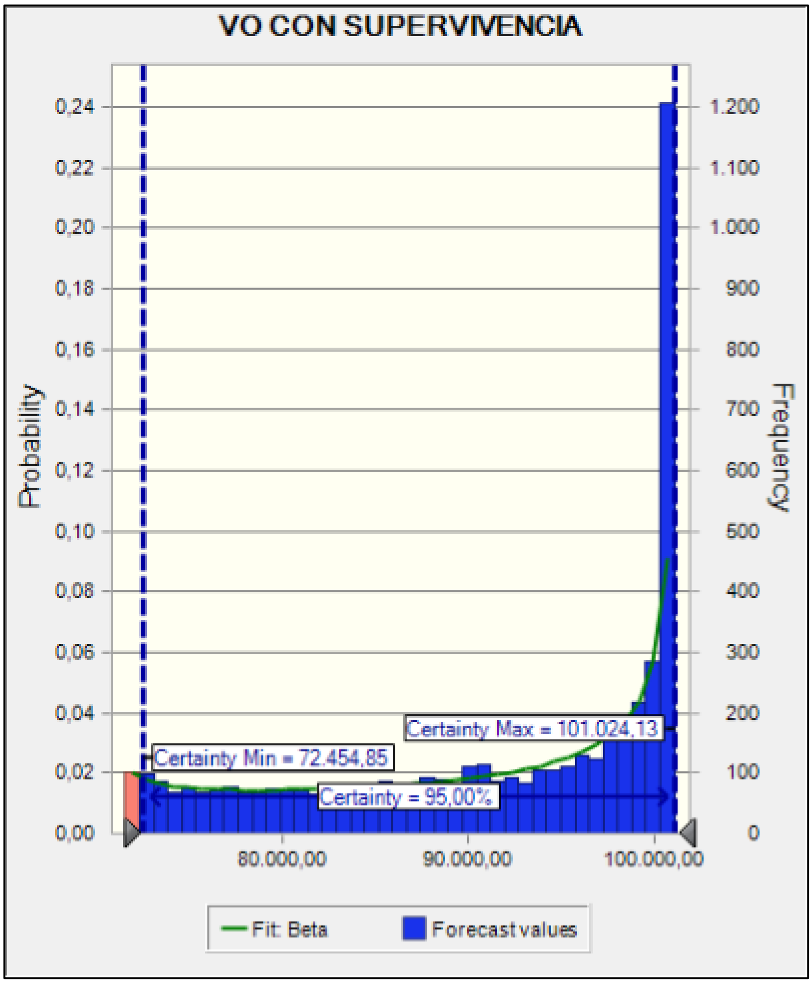
| | |
|---|---|
| Objectives | 1 |
| Requirements | 0 |
| Constraints | 0 |
| Linear | 0 |
| Non-linear | 0 |
| Constant | 0 |
| Assumptions | 1 |
| Correlations | 0 |
| Correlation matrices | 0 |
| Decision variables | 2 |
| Forecasts | 2 |

Annex IV

**Worksheet: [MODESTOCASTICO.xlsx]Sheet1**

**Forecast: VO CON SUPERVIVENCIA**

Summary:
Certainty level is 95,00%
Certainty range is from 72.454,85 to 101.024,13
Entire range is from 71.531,42 to 101.025,79
Base case is 80.654,65
After 5.000 trials, the std. error of the mean is 132,73




**Forecast: VO CON SUPERVIVENCIA (cont'd)**

| Statistics: | Forecast values | Precision |
|---|---|---|
| Trials | 5.000 | |
| Base Case | 80.654,65 | |
| Mean | 91.406,11 | 0,28% |
| Median | 94.408,49 | 0,51% |
| Mode | --- | |
| Standard Deviation | 9.385,41 | 1,43% |
| Variance | 88.085.926,41 | |
| Skewness | -0,6493 | |
| Kurtosis | 2,06 | |
| Coeff. of Variation | 0,1027 | |
| Minimum | 71.531,42 | |
| Maximum | 101.025,79 | |
| Range Width | 29.494,37 | |
| Mean Std. Error | 132,73 | |

| Percentiles: | Forecast values | Precision |
|---|---|---|
| 0% | 71.531,42 | |
| 10% | 76.133,01 | 0,49% |
| 20% | 81.529,56 | 0,74% |
| 30% | 86.313,90 | 0,64% |
| 40% | 90.447,57 | 0,43% |
| 50% | 94.407,01 | 0,51% |
| 60% | 97.564,10 | 0,32% |
| 70% | 99.478,15 | 0,20% |
| 80% | 100.674,86 | 0,08% |
| 90% | 101.003,66 | 0,00% |
| 100% | 101.025,79 | |

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**Notes**

Note 1. It is assumed constant, but it could be variable over time, and even stochastic as well.

Note 2. It could be time variable like (g).

Note 3. See in the annex greater detail of the feasible solutions found in the stochastic optimization.