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Original Paper

Actuarial Assumptions on Contingent Output Rates under Multidecrements

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Abstract

In the **actuarial valuation of "severance-type" defined benefit plans** where a fraction of the total benefit is recognized, due to death, resignation or dismissal, in addition to retirement, it is very important to define the models of the underlying rates of the experience of the company, on the issue of payment for resignations and dismissals. The demography in terms of **the distribution of personnel by age,** assuming different models, **negative exponential, linear and constant,** is indicative of the actuarial cost and Liabilities. A higher or lower cost **will depend on the model used** and the **distribution of the company's personnel by age.** The latter, although evident, can be complicated in terms of the Assumptions and Hypotheses that are used.

Keywords: IAS 19, Actuarial Liabilities, Layoffs, Resignations, dismissals, Multi-decrements, Projected Benefit Method.

1. Aim

The objective of this paper is to assess the **impact of the Actuarial Liabilities** in the same population, using **three different models** for exits due to resignations, which can differ significantly, in the generation of different liabilities, equally valid depending on the model chosen in the hypotheses.

2. Motivation of the Paper

It is very common for actuaries and auditors to discuss actuarial valuation Assumptions and Hypotheses. Some definitely believe that the methodology to arrive at some hypotheses **are absolute truths** and in our humble opinion nothing **could be further from the truth**. It is true that in the 1st world there is generally **relative dynamic stability of** corporate and government sovereign bond yields. Similarly, inflation control and salary increase rates are relatively easy to set. In addition to all of the above, there are sources of information that support the choice of one particular assumption or another with macroeconomic indices, the interest rate to discount the obligations, is one of them.

In emerging countries, read in much of Latin America and specifically in Venezuela with hyperinflation, everything summarized above within the context of the 1st world **is not necessarily true and it is important to know the country's economy and its macroeconomic distortions well.**

Therefore, it is not strange to obtain valuations of the same contingency with different assumptions and probably all of them valid, depending on the final choice of the hypotheses and how they were based.

However, the case at hand **is not so much about economic assumptions or hypotheses financial** but rather demographic. Specifically, the adjustment of models to the historical annual rates, (averages observed in the statistics of personnel exits and specifically to the benefits for resignations) which is the contingency that we will study here.

Job instability in these Latin American countries does not compare with the relative job stability of the 1st world.

3. Sample Definition

a. It is a Latin American company with the following historical experience of departures from 2018-2022.

Table 1.

HISTORIC OBSERVED EXITS						
Exits						**Total**
	2018	**2019**	**2020**	**2021**	**2022**	
Pension	0	0	0	0	0	0
Death	0	0	0	3	6	9
Dissmisal	200	269	207	73	547	1296
Resignation	426	554	227	603	760	2570
Others	50	131	51	110	179	521
TOTAL	**676**	**954**	**485**	**789**	**1.492**	**4.396**

EXITS RATES						
Pension	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%
Death	0,00%	0,00%	0,00%	0,38%	0,40%	0,20%
Dissmisal	29,59%	28,20%	42,68%	9,25%	36,66%	29,48%
Resignation	63,02%	58,07%	46,80%	76,43%	50,94%	58,46%
Others	7,40%	13,73%	10,52%	13,94%	12,00%	11,85%
TOTAL	**100,00%**	**100,00%**	**100,00%**	**100,00%**	**100,00%**	**100,00%**

TOTAL POPULATION YEAR 2022=4242

Table 2.

BENEFITS	**2018**	**2019**	**2020**	**2021**	**2022**	
Dissmisals	200	269	207	73	547	1296
Initial Populations	3614	3414	3671	3621	4196	18516
Pension/Population	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%
Death/Population	0,00%	0,00%	0,00%	0,08%	0,14%	0,05%
Dissmisals/Populatioı	**5,53%**	**7,88%**	**5,64%**	**2,02%**	**13,04%**	**7,00%**
Resignation	**11,79%**	**16,23%**	**6,18%**	**16,65%**	**18,11%**	**13,88%**
Other Popular	1,38%	3,84%	1,39%	3,04%	4,27%	2,81%
Total Population	18,71%	27,94%	13,21%	21,79%	35,56%	23,74%

Assumption Departures						
Pension Rate						
Death Rate	100%	100%	100%	100%	100%	100%
Dissmisal Rate	**5,53%**	**7,88%**	**5,64%**	**2,02%**	**13,04%**	**7,00%**
Resignation Rate	**11,79%**	**16,23%**	**6,18%**	**16,65%**	**18,11%**	**13,88%**

Payments are only made in the event of death, resignation and dismissal. Other causes of exits are not remunerated in payment. **In this work, only resignations are going to be modeled,** the other contingencies would be carried out in the same way, especially layoffs. Generally, in the case of mortality, a table already in use such as GAM83 or any other is used.

b. In addition to the departures profile, the distribution of current staff by age is also available.

Without loss of generality and only to simplify the calculations and show the results as pedagogical as possible, exit models due to resignation are determined in a unisex manner.

Company Demographic Profile

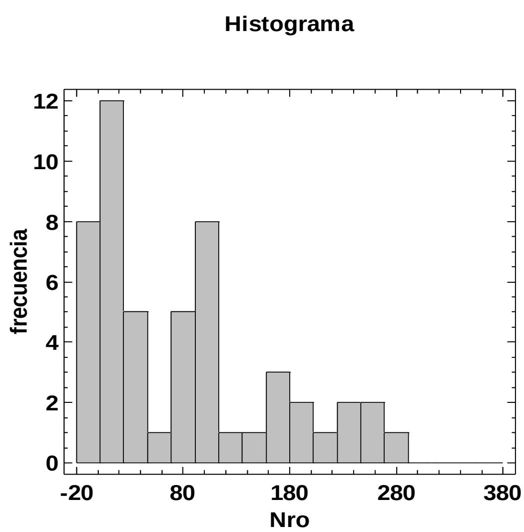
Table 3.

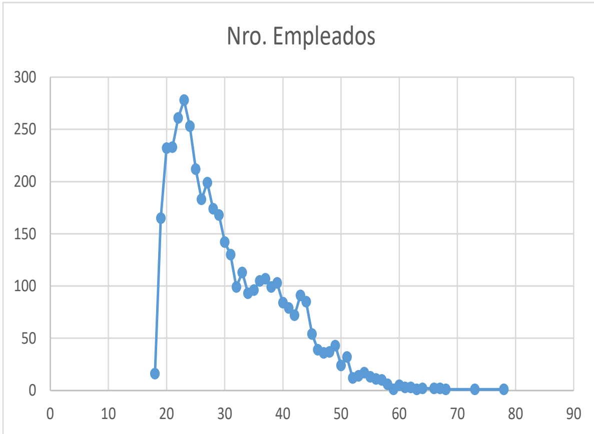
AGE	#Employee	AGE	#Employee
18	16	46	39
19	165	47	36
20	232	48	37
21	233	49	43
22	261	50	24
23	278	51	32
24	253	52	12
25	212	53	14
26	183	54	17
27	199	55	13
28	174	56	11
29	168	57	10
30	142	58	6
31	130	59	1
32	99	60	5
33	113	61	3
34	93	62	3
35	96	63	1
36	105	64	2
37	107	66	2
38	99	67	2
39	103	68	1
40	84	73	1
41	79	78	1
42	72		
43	91		
44	85		
45	54	Total general	4242

AGE		*#Employees*	
Mean	43,826923	Mean	81,57692
Standard Error	2,1844993	Standard Error	11,3482
Median	43,5	Median	63
Mode	#N/D	Mode	1
Standard Deviation	15,752648	Standard Deviation	81,83301
Sample Variance	248,14593	Sample Variance	6696,641
Kurtosis	-0,949804	Kurtosis	-0,29841
Skewness	0,1450388	Skewness	0,882497
Range	60	Range	277
Minimum	18	Minimum	1
Maximum	78	Maximum	278
Sum	2279	Sum	4242
Count	52	Count	52
Confidence Level(95,0%)	4,3855653	Confidence Level(95,0%)	22,78245

No.: # employees

The distribution of staff by age is shown below.





4. Output Modeling

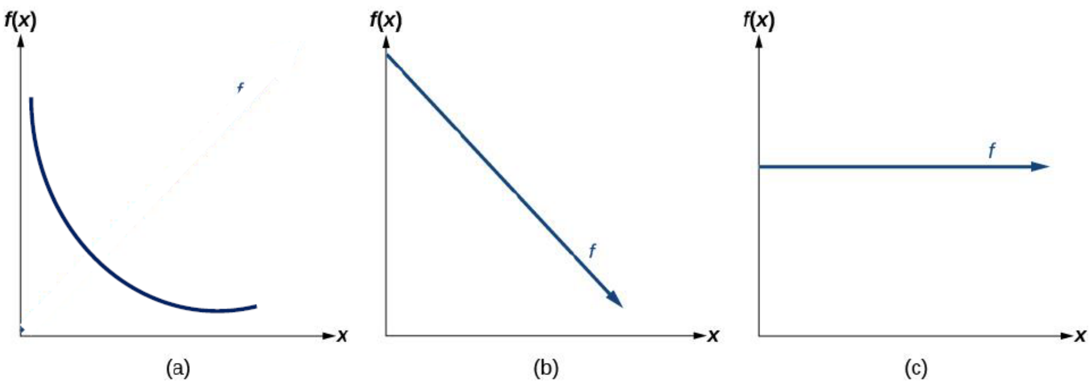
Below is a relatively simple description of the models that will be used to measure the impact of exits by age.

In this case, 3 different elementary models will be evaluated. Generated by the observed rates of the fiscal period of the valuation.

a. Exponential model (ME) parameter θ.

b. Linear Model (ML) parameter (a, b).

c. Constant Model (MC) $K =$rate of uniform departures



$f(x)$: *Resignations Rates, Age of employees*

a. Exponential Model:

Industrial Relations theory supports the idea that a young employee generally has a higher exit rate than an older one. This is a **universally accepted and easily understood trend.** In the case of layoffs, although the underlying concept is not exactly the same, it is also relatively well accepted; that the company generally dismisses personnel who for some reason are not suitable at an early age and does not wait for considerable seniority to accumulate in time to dismiss them due to the cost **of the individual's non-productivity.**

Obviously, there are a number of potential causes for dismissal that could eventually deviate from this trend a bit, but the truth is that it ends up being accepted, probably under regular company conditions, which in effect ends up observing more dismissals at younger ages.

Based on the sample statistics of resignations, an $\theta = 0{,}05$ initial parameter is defined and it is sensitized so that the quotient between the exits generated by this parameter, and the exits due to resignations observed in the demographic profile of the company are equal.

The function $f(x) = e^{-\theta x}$defines for all the ages of the profile from $x = 18$ $x = 78$the theoretical rates in such a way that the exit rate due to resignation generated by $f(x)$is the same as that verified by the historical statistical experience of the company, that is, it is about finding a

$$\{\theta \mid \#Modeled\ Exits\ /\ población\ total\} = \{Resiguations\ Historic\ observed\ rate\}$$

$opt\left\{\frac{\tau^e}{\tau^b} = 1\right\}$ where:

τ^h: $Resignation\ historic\ rate$

τ^e: $simulated\ rate\ with\ f(x \mid \theta)\ (x, para\ x \in [18{,}78])$

l_x: $\#\ employes\ of\ age\ x$

By applying the previous model to the distribution of personnel for the corresponding fiscal year, we obtain the following:

The historical resignation rate of the company, 13.87%, coincides with that generated by $f(x \mid \theta)$when it is applied to the profile of the company's personnel. When $\theta = -0{,}0704$, the differential quotient of both the real and modeled rates is unity.

Table 4.

RESIGNATION	0,13879888	ALPHA	-0,07041596
POPULATION	4242	DIFERENTIAL	1,0000869
EXITS	588,733672		0,138786816

AGE	#	RESIGNATIC	EXITS
18	16	0,28153814	4,504610299
19	165	0,26239526	43,2952172
20	232	0,24455397	56,73652062
21	233	0,22792578	53,10670677
22	261	0,21242821	55,4437623
23	278	0,19798438	55,03965681
24	253	0,18452264	46,68422766
25	212	0,17197622	36,45895811
26	183	0,16028288	29,33176644
27	199	0,14938461	29,72753784
28	174	0,13922736	24,22556129
29	168	0,12976075	21,79980543
30	142	0,1209378	17,17316789
31	130	0,11271476	14,65291935
32	99	0,10505084	10,40003348
33	113	0,09790802	11,06360653
34	93	0,09125087	8,486330917
35	96	0,08504636	8,164450925
36	105	0,07926373	8,322691267
37	107	0,07387427	7,904547235
38	99	0,06885127	6,816275754
39	103	0,0641698	6,609489507
40	84	0,05980664	5,023758043
41	79	0,05574015	4,403472165
42	72	0,05195016	3,740411609
43	91	0,04841787	4,406025719
44	85	0,04512574	3,835688204
45	54	0,04205747	2,271103187
46	39	0,03919781	1,528714728
47	36	0,0365326	1,31517359
48	37	0,0340486	1,259798359
49	43	0,03173351	1,364540736
50	24	0,02957582	0,709819665
51	32	0,02756484	0,882074972
52	12	0,0256906	0,308287207
53	14	0,0239438	0,335213135
54	17	0,02231576	0,37936796
55	13	0,02079843	0,270379535
56	11	0,01938426	0,213226847
57	10	0,01806625	0,180662467
58	6	0,01683785	0,101027108
59	1	0,01569298	0,015692979
60	5	0,01462595	0,07312976
61	3	0,01363148	0,040894428
62	3	0,01270462	0,038113854
63	1	0,01184078	0,011840781
64	2	0,01103568	0,022071359
66	2	0,00958598	0,019171962
67	2	0,00893419	0,017868384
68	1	0,00832672	0,008326721
73	1	0,00585555	0,00585555
78	1	0,00411776	0,004117763
Grand Total	4242		588,7336724

By optimizing the exponential model and zeroing out the difference between $(\Delta\tau = 0)$or equivalently $\left(\frac{\tau^e}{\tau^h} = 1\right)$, , the resignation rates that emulate the exits of personnel equal to the annual resignation observed rate.

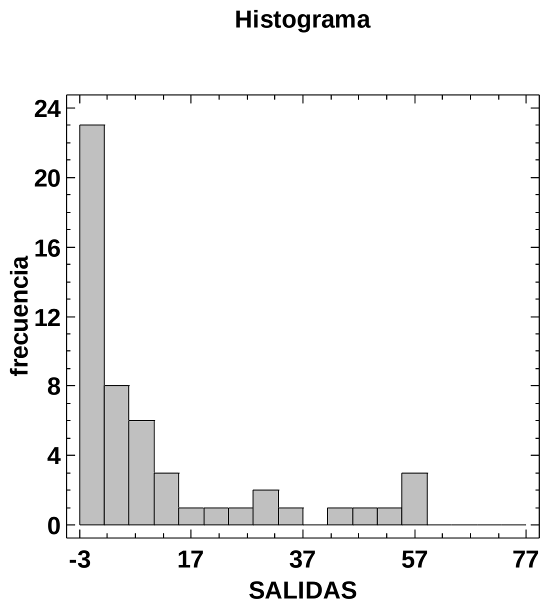
The descriptive statistics of the outputs are as follows:

From the table it is easy to see that the total number of departures due to resignation generated by the exponential model is 588.73.

a. Exponential Model

Table 5.

RESIGNATIONS %		# OF EXITS	
Mean	0,0774	Mean	11,3218
Standard Error	0,0105	Standard Error	2,3699
Median	0,0468	Median	3,7880
Mode	#N/D	Mode	#N/D
Standard Deviation	0,0756	Standard Deviation	17,0896
Sample Variance	0,0057	Sample Variance	292,0552
Kurtosis	0,4565	Kurtosis	1,5693
Skewness	1,2019	Skewness	1,6739
Range	0,2774	Range	56,7324
Minimum	0,0041	Minimum	0,0041
Maximum	0,2815	Maximum	56,7365
Sum	4,0262	Sum	588,7337
Count	52,0000	Count	52,0000
Confidence Level(95,0%)	0,0210	Confidence Level(95,0%)	4,7578



From the histogram it is easily observed that the highest density of staff exits, as expected, is at young ages and with less intensity in the oldest.

b. Linear Model

The linear model, like the exponential one, is a decay model that reflects the same theoretical principle outlined above. The only difference is that this decay is not so fast and the slope of the line of the linear model of adjustments is of course only negative.

The function $f(x \mid a, b] = a - bx$ is a linear model that optimally fits the data. When applied $f(x \mid a, b)$ to the same demographic profile, the goal is to find the same exit rate observed historically. In this case, the vector of parameters to be optimized is:

$$\left\{\theta = (a,b) \mid \frac{\tau^h}{\tau^e} = 1\right\}$$
$$opt\left\{{\tau^e}/{\tau_h} = 1\right\}$$

In the case of the data at hand $\theta = (a, b)$and the results originated by $f(x \mid a, b)$ meet the objective.

a= 25.35397

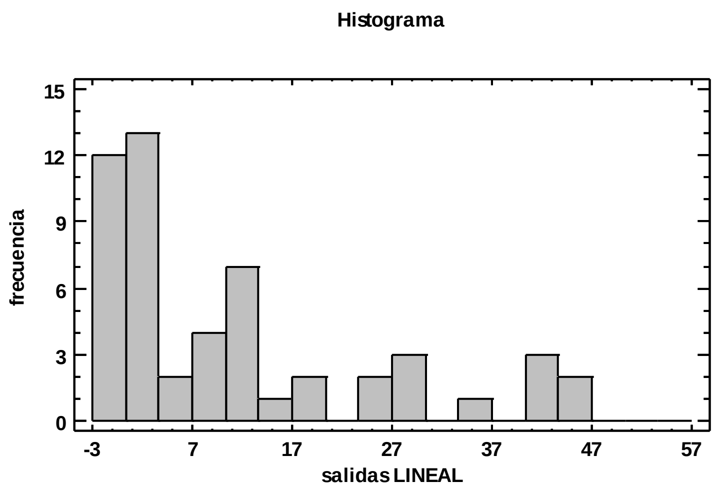
b=0.376115

Table 6.

EDAD	Nro	RENUNCIA	SALIDAS
18	16	0,185839	2,973424
19	165	0,18207785	30,04284
20	232	0,17831669	41,36947
21	233	0,17455554	40,67144
22	261	0,17079439	44,57734
23	278	0,16703324	46,43524
24	253	0,16327208	41,30784
25	212	0,15951093	33,81632
26	183	0,15574978	28,50221
27	199	0,15198863	30,24574
28	174	0,14822747	25,79158
29	168	0,14446632	24,27034
30	142	0,14070517	19,98013
31	130	0,13694402	17,80272
32	99	0,13318286	13,1851
33	113	0,12942171	14,62465
34	93	0,12566056	11,68643
35	96	0,12189941	11,70234
36	105	0,11813825	12,40452
37	107	0,1143771	12,23835
38	99	0,11061595	10,95098
39	103	0,1068548	11,00604
40	84	0,10309364	8,659866
41	79	0,09933249	7,847267
42	72	0,09557134	6,881136
43	91	0,09181019	8,354727
44	85	0,08804903	7,484168
45	54	0,08428788	4,551546
46	39	0,08052673	3,140542
47	36	0,07676558	2,763561
48	37	0,07300443	2,701164
49	43	0,06924327	2,977461
50	24	0,06548212	1,571571
51	32	0,06172097	1,975071
52	12	0,05795982	0,695518
53	14	0,05419866	0,758781
54	17	0,05043751	0,857438
55	13	0,04667636	0,606793
56	11	0,04291521	0,472067
57	10	0,03915405	0,391541
58	6	0,0353929	0,212357
59	1	0,03163175	0,031632
60	5	0,0278706	0,139353
61	3	0,02410944	0,072328
62	3	0,02034829	0,061045
63	1	0,01658714	0,016587
64	2	0,01282599	0,025652
66	2	0,00530368	0,010607
67	2	0,00154253	0,003085
68	1	-0,0022186	-0,002219
73	1	-0,0210244	-0,021024
78	1	-0,0398301	-0,03983
Total general	4242		588,7848

Table 7.

RENUNCIA		SALIDAS	
Mean	0,0887	Mean	11,32279
Standard Error	0,008216	Standard Error	1,930174
Median	0,08993	Median	5,716341
Mode	#N/D	Mode	#N/D
Standard Deviation	0,059248	Standard Deviation	13,91868
Sample Variance	0,00351	Sample Variance	193,7296
Kurtosis	-0,9498	Kurtosis	0,399725
Skewness	-0,14504	Skewness	1,251993
Range	0,225669	Range	46,47507
Minimum	-0,03983	Minimum	-0,03983
Maximum	0,185839	Maximum	46,43524
Sum	4,6124	Sum	588,7848
Count	52	Count	52
Confidence Level (95,0%)	0,016495	Confidence Level (95,0%)	3,874985



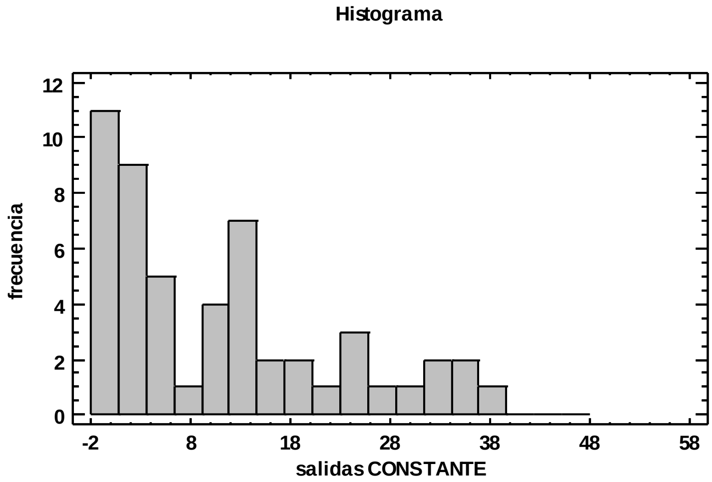
This histogram clearly shows that, although the highest density of departures is also at young ages, a significant number of departures begin to appear at intermediate ages.

c. Constant Model

This is a trivial model, sometimes used by some colleagues, assuming uniformity of rates for all ages, which obviously **violates the aforementioned principle.** The foregoing does not invalidate the model at all, but once again, it would not be in line with the duty of resignation behavior. It would be rare for a company to have the same rate for all ages.

The function $f(x) = k$ for all x, describes that the resignation outputs by age are the same for all ages and equal to the global annual rate of the company.

In this case there is no parameter to optimize/raise awareness, this rate is simply applied to the number of employees for all ages. **Obviously the $\tau^h = \tau^e$ by construction.**



In this, the outputs correspond to the population density at each age.

5. Comparative Analysis of the Results (Note 1)

Below are the expected values of the 1st year liability, without adjusting for interest. This term is the largest of the series of terms for the single premium of a mixed quasi-insurance. The comparison made with this term defines in a certain way which model is more or less expensive.

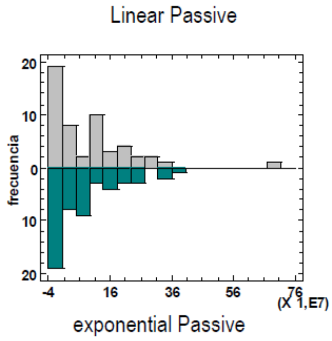
Table 8.

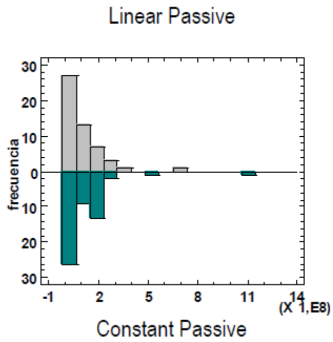
LINEAL MODEL LIABILITY		CONSTANT MODEL LIABILITY		EXPONENTIAL MODEL LIABILITY	
Mean	95914707,67	Mean	110302924	Mean	78855883,61
Standard Error	17744218,7	Standard Error	24198403,64	Standard Error	13977562,18
Median	38416075,47	Median	56149533,88	Median	34906233,84
Mode	0	Mode	#N/D	Mode	#N/D
Standard Deviation	127955380,7	Standard Deviation	174497170,2	Standard Deviation	100793634,3
Sample Variance	1,63726E+16	Sample Variance	3,04493E+16	Sample Variance	1,01594E+16
Kurtosis	9,140007199	Kurtosis	21,86274106	Kurtosis	0,830645551
Skewness	2,433465169	Skewness	4,080985332	Skewness	1,302041432
Range	708139545,2	Range	1116297970	Range	362926414,6
Minimum	0	Minimum	1157,815454	Minimum	34,34902008
Maximum	708139545,2	Maximum	1116299128	Maximum	362926449
Sum	4987564799	Sum	5735752047	Sum	4100505948
Count	52	Count	52	Count	52
Confidence Level(95,0%)	35623005,48	Confidence Level(95,0%)	48580322,41	Confidence Level(95,0%)	28061126,98

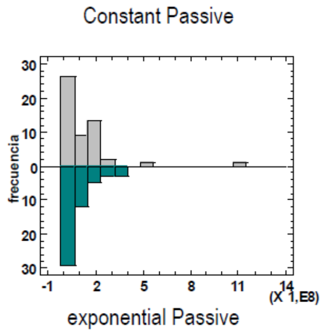
LIABILITY SUMMARY

1	LINEAL	4.987.564.798,98	87,0%
2	CONSTANTE	5.735.752.046,99	100,0%
3	EXPONENCIAL	4.100.505.947,76	71,5%
	TOTAL	14.823.822.793,73	

Following box- plots:





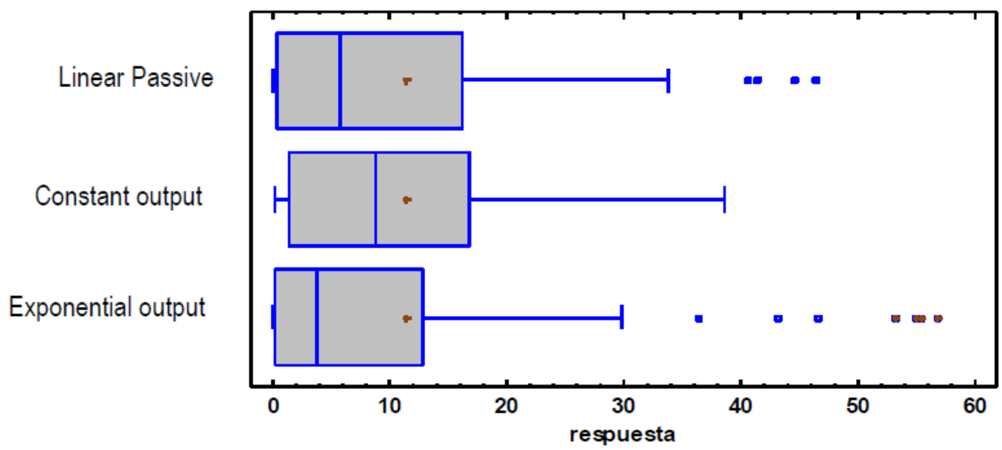


i. The exponential model seems to concentrate the data more with some outsiders in the upper tail.

ii. The constant and linear models have, in principle, greater volatility (Note 2).

As can be seen from the comparative analysis of withdrawals due to resignation, generated by each model, the actuarial liabilities expressed in terms of expected value are quite different, although all of them generate the same # of withdrawals due to resignation.

Falling into the temptation of saying which is better is absurd, it will depend on the experimental cloud of exits by age and seek the best possible fit.



6. Conclusions

From the entire **unisex quit rate adjustment analysis,** the following conclusions can be drawn:

a. The adoption of one or the other model will depend on which of them best reflects the resignations in the company and therefore, in addition to having a group of annual historical data on departures, it is insufficient. **A formal study of this contingency should be made by age and in some cases, bivariate of an analysis, the age and the years of service to which they resign.**

b. In practice, it could be the case that the experimental point cloud of the exits due to resignation are very dispersed and/or are concentrated in a reduced age group, forming some type of cluster; In that case, the ideal is probably to make cuts and adjust ladder-type rates; where the highest percentage of resignations are concentrated in certain ages.

c. Another no less relevant aspect is the exit rates for high ages. In any case, generally the rates of resignation, dismissal and death, given a model culminate in advanced ages in very small rates, for all the explanations that we have indicated in this work, therefore, it is necessary to define at the end, the outputs of the company in such a way that the sum of them obviously equals one, at the exit threshold age. This is usually done with a final rate at the threshold age with a 100% exit probability. **Another alternative would be to set exit rates for each contingency at that threshold age so that the total sum is also 100%.**

d. This last reflection of setting an exit rate for each contingency may seem a bit arbitrary, in the sense that, if the exponential or linear model is used, certainly at high ages, the exit probability is zero, very close to it. Considering an exit benefit, for example, of waiving the threshold age of 13.87%, when exit rates in the order of immediately previous ages are recorded, looks 10^{-6}a bit strange. **The above should simply be understood as a formal output assignment.**

e. When auditing the actuarial valuation, in this type of benefits, it must be taken into account that if there is no prior information on how the departures are by age, sex and seniority, probably the uniform model of equality of departures by sex, age and seniority is the best, since its Liability has the greatest impact. Notwithstanding the foregoing, if the demographic profile of the company, when classified by completed age, is concentrated in young ages, the uniform rate of departures due to resignation would probably be underestimating them, since it would be lower than that of the other 2 models and very particularly to the exponential. That is why a model should not be assigned without greater detail of key and critical data.

f. Last but not least, it is that all the analysis of the output models underlying the history and statistical experience up to the year of valuation is known. It is very likely that, in the future, the output behavior will be different. For example, history indicates a linear model, but it is possible that this model will migrate to an exponential in the next fiscal period.

The foregoing is an indication that there may be very different liabilities for a contingent valuation based not only on history but also on the immediate future prospects of the actuary. Therefore, it should be avoided in audits making lighthearted value judgments, which bias the validity of one study over another and personally we consider that such judgment can be wrong.

In the example that concerns us, as can be seen in the table of the expected values of the liabilities, the exponential model generates a cost or expected value of 1 year in the order of 71.5% and the linear one 87.0%. on the uniform model.

g. As a great conclusion, actuarial valuations of the same sample of employees with different costs and liabilities can coexist. Affirming categorically that one or the other is the most appropriate involves analyzing the point cloud of employee departures for each contingency. However, the future is uncertain and it is also possible that for a next fiscal period the previous experience will not be maintained.

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Notes

Note 1. Liabilities in terms of the expected value of the 1st year without adjustment for interest. Generally, the total liability corresponds to the calculation of the expected present value, which in the nomenclature of a mixed insurance would be $A_{x:r-x} + \frac{D_r}{D_x}$

Note 2. Without taking into account the outsiders of the upper tail.