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Original Paper

The Extrapolations in Forecasting: A Function of Associational-Effects?

Gina Holton[1] & Edward J. Lusk[2]

[1] Department of Supply Chain Management and International Business; School of Business and Economics, SUNY: Plattsburgh, USA

[2] Emeritus: The Wharton School, Dept. Statistics, The University of Pennsylvania, USA & School of Business and Economics, SUNY: Plattsburgh, USA & Chair: International School of Management: Otto-von-Guericke, Magdeburg, Germany

[*] Correspondence: lusk@wharton.upenn.edu or luskej@plattsburgh.edu

Abstract

Context According to Tamhane & Dunlop (2000, p. 363) [T&D] re: ***Forecasting***:

> *"- - -extrapolation beyond the range of the data is a* **risky business** *should be avoided."*

If Tamhane & Dunlop are correct that: Forecasting Extrapolations somehow compromise the linkages of the Projection of the Past into a Relevant Future, ***Then,*** indeed, Extrapolations should be avoided. This *musing* begs a Question: *What is the Nature of the Jeopardy if forecasters routinely elect Extrapolations to inform their decision-making processes?* ***Focus*** To provide information on this pivotal-question, we randomly selected Firms listed on the S&P$_{500}$ to test the Impact of Extrapolations on forecasting profiles. ***Inferential Design Issues*** As the S&P$_{500}$ Firm-Data-Panels are, in the main, driven by multiplicative- processes, we made the following decisions: The critical forecasting variables of interest to be profiled re: the impact of Extrapolations are: The Capture Rate of the 95%Prediction Interval & The Benchmarked-Precision. We will examine the **Ext**rapolation & **Int**erpolation Effects for four Exhaustive cases: {[Y]Int & [X]Int : [Y]Int & [X]Ext : [Y]Ext & [X]Int : [Y]Ext & [X]Ext}. ***Results*** There are clear inferential-indications that **Ext**rapolations in the X-domain do indeed compromise the theoretical profiles that one may expect—to wit: The Failure to Capture Rate of the 95%Prediction Interval, for such X-Extrapolations, ***was on the order of 50%—far below the 95% theoretical expectation***. Additionally, the Benchmarked-Precision of the 95%Prediction Interval for these X-***Extrapolations*** was inferentially ***wider—much less precise than expected***. We discuss the impact of this result on the development of forecasting protocols.

Keywords: Forecasting Model Development, Extrapolation-Jeopardy

1. Introduction

1.1 Overview

Forecasting-Models and their Protocols are ubiquitous in the panoply of decision-makers because—***IF there are No reliable projections of the likely future, then random chance will be the decision-making GPS!*** Thus, forecasting, usually using standard statistical models, is an essential analytic-activity in all organizations. However, surprisingly, our experiential evidence is that there is a **conceptual** ***thorn*** in the Forecasting-***Rose*** that beg the following research question:

What is the Magnitude of the Effect on Decision-Making Intel of using ***Extrapolations in Creating Forecasts?***

This question was motivated by the following comment offered by Tamhane & Dunlop (2000, p. 363):

> *"- - -extrapolation beyond the range of the data is a* **risky business** *should be avoided."*

2. Our Research Montage: The Components

To address our question of interest, and thus provide information intended to inform the forecasters, we offer a research design, the components of which are:

2.1 The Forecasting Model

Assume that we have the standard OLSR-Forecasting Model:

Forecast[X] ≡ [Y_X = [$\hat{\alpha}$ + $\hat{\beta}$ × X_{PPV}] Eq[1]

where: $\hat{\alpha}$ is the Intercept; $\hat{\beta}$ is the Slope of the Two-Parameter Linear Forecasting Model; these population-estimates of the Intercept- and Slope-parameters are determined by minimizing the **O**rdinary **L**east **S**quares of the Cartesian-Profile of the n-Pair of{Y🡐X}-points that are a random sample from a target population, and $X_{proposed}$ is the $X_{Proposed}$-*Projection-**Value*** [X_{PPV}] selected by the analyst to produce the forecast {Y_X}.

2.2 The 95% Prediction Interval[95%PI]

The center of the 95%PI is the projected-value of Y_X. The 95%PI assumes that:

I. the n-Pair of{Y🡐X}-points used to create the Forecasting Model [Eq[1] as well as the 95%PI are a random sample of sufficient size ***from the population of interest***, where the Central Limit Theorem will apply for the sampling protocol,

II. the X_{PPV} is IN the ordered-interval of X-Values use to create the OLSR Model [Eq[1]]—i.e., X_{PPV} is an Interpolation, and

III. ***if*** conditions I & II are satisfied, ***then and only then*** (i) this particular 95%PI has a 95% chance of containing the "*Next to be observed Y-value* [Y_{Next}]" somewhere ***IN*** this particular 95%PI, and (ii) also, this particular 95%PI has a 5% chance of ***NOT*** containing the "*Next to be observed Y-value* somewhere IN this particular 95%PI.

2.3 The Definition of an Extrapolation

A Forecasting Extrapolation occurs when *an [X_{PPV}] that is **NOT IN** the ordered-interval of the X*-values *used to create The Forecasting Model* [*Eq*[*1*]] *is used to create*: [*i*] *a Forecast* [Y_X], as well as [*ii*] *the related* 95%PI. ***Critical Condition** In an inferential-context, a Forecasting Extrapolation is considered a **Calculation Error**, the nature of which **likely will compromise** the decision-making relevance of* [*i*] *the Forecast* [Y_X], as well as [*ii*] *the related* 95%PI.

2.4 Research Measures

Our two decision-making measures of interest are:

I. The ***Capture Rate*** of the 95%PI for the [Y_{Next}] "*Next to be observed Y-value", and*

II. The ***Width of the Benchmarked Precision*** of the 95%PI.

2.4.1 The Width of the 95%PI

An obvious and critical measure in most all forecasting investigations is the Capture Rate of the 95%PI. However, a related question is almost never ever broached. *What is the effect of Extrapolations on the **Width of the 95%PI?*** The reason for this seems to be the "Leap of Inferential-Faith" that the 95%PI performs as advertised sort of: *no matter what the forecasting protocol happens to be*! In this regard, Tamhane & Dunlop (2000, p.363) offer the following skeptic's alert ['*Paraphrasing'*]:

'*Both the 95%CI & 95%PI have the shortest widths when the X_{PPV} is equal to $\bar{x}$*:[*The Mean of the X-Panel used to create the OLSR Model*]. *The intervals around both the 95%CI & the 95%PI become parabolically- & symmetrically-wider as the X_{PPV} deviates from $\bar{x}$ on either side. - - -. For **Extrapolations**-i.e., X_{PPV}-values: (i) outside the ordered-interval of the X-values used to create the OLSR-Model and (ii) "**far away"** from $\bar{x}$, it can no longer be **expected** that the Capture Rates of 95%PI will conform to the mathematical-expectation assumed for the 95%PI—**because the OLSR-Linear Model may not hold at these extreme points**.'*

2.4.2 Benchmarking the Precision

Given these Extrapolation musings of T&D, we added, in addition to the Capture Rate, an inferential component to examine for the first time, the effect of Extrapolations on the Width of the 95%PIs. This measure is developed following. The ***Precision*** of the 95%PI is:

[The Upper Limit of the 95%PI – The Lower Limit of the 95%PI] / 2

However, the ***Precision*** of the 95%PI is impacted by the Measure of the Variables under examination. Thus, to facilitate inferential analyses, it is standard to benchmark Precision by the Mean or the Median of the Y-Variate used to create the OLSR Model. We prefer the Median as it is less sensitive to asymmetry-effects. Thus, our Precision test-Variable will be;

The **B**ench**M**arked **P**recision: [Precision[95%PI] / Median[Y-Variate[Panel]]

or simply: **BMP**[95%PI].

3. Research Measures & Variables of Inferential Interest

3.1 Classification Taxonomy

For the two Variables of Interest, we have formed the following logical and exhaustive Classification Table to collect sample-intel on the nature of the effect of Extrapolations on the **Capture Rate** for the 95%PI and also to address the related **Width** of the BMP[95%PI]:

Table 1. A Screening Taxonomy for **Associational-Driven Y & X[Variates]**

For a Single 95%PI	$Y_{X_{PPV}}$ [Ext]	$Y_{X_{PPV}}$ [Int]
X_{PPV}[Ext]	Questionable Inferential Value [**X]Ext & [Y]Ext**	Suggests **[X]** Investigative Alert **[X]Ext & [Y]Int**
X_{PPV}[Int]	Suggests **[Y]** Investigative Alert [**X]Int & [Y]Ext**	Inferential Gold Standard **X[Int[& [Y]Int**

3.1.1 Overview Context

We will be selecting Market Trading Panels from the S&P$_{500}$ so as to demonstrate the Extrapolation-Associational-effects on: (i) The Capture Rate of the 95%PI & (ii) The related Width-issue under four exhaustive-conditions of Table 1. This sampling context is very important relative to the performance of the Associational-effects re: ***The Capture Rate*** & ***The Width of the 95%PI*** for these four Cells.

3.1.2 The Associational Effect

Trading Market Panels are almost exclusively *individually* driven by *associational-generating processes*[i]. ***Expected Effect*** There very likely will be longitudinal-associations individually for the {Y🡐X}-Panels. Thus, the Pearson Product Moment Correlation [PPMC] for [[Y ***OR*** X] with a Standard Time-Index[ii] [1, - - -, n] may be Positive or Negative and will likely be strong-enough to reject the standard False Positive Error FPE[Null]. In this case, there will be various associational-effects that will need to be either (i) integrated into a DOE-inferential-testing frame, or (ii) controlled as inferential-blocking-variables. We opted for the latter as it greatly facilitates the exposition and understanding of the Effect of Extrapolations for the four-Profiles of Table 1. ***Clarification*** For example, if there is low PPMC-association for the {Y & X}-variates with the Time-Index, e.g.,≈0, then concern over Extrapolations is very likely to be inferentially-moot as the Panel-Values selected as X_{PPV}[Ext] & the related [Y_{Next}], will ***likely*** be draws from the same Population from which the {Y & X}-variates were selected to fit the OLSR-Model. Alternatively, if there is strong positive PPMC-association for the {Y & X}-variates, e.g., for example ≈0.35, then

X_{PPV}[Ext] will rarely have an associated Y_{Next} that is IN the 95%PI formed from the {Y & X}-variates used to fit the OLSR Model as the associational-profile will "*push*" them outside the Range of the {Y & X}-variates used to fit the OLSR Model. Thus, for inferential testing for the Effect of Extrapolations for the S&P$_{500}$, we have decided to use the following Blocking- or Selection-Screens for qualifying the firms for testing:

I. ***Y-Variate Selection Screen*** We will randomly select 60 firms from the S&P$_{500}$.Their Y-Panels will only be used in inferential testing if: (i) their PPMC with The Unit-Time Index] was Positive, and (ii) was ≥ 0.35. We judged this to be evidence of a relatively strong associational-generating process for the Y-variate, and

II. ***X-Variate Selection Screen*** So as to ***permit*** the opportunity to have variation in the profiles noted in Table 1, we did not conditionally select the X-Drivers based upon their associational-profiles. ***Results*** In this regard, the [PPMC[*Stock Value* (*w.*) *The Time Index*]] was on Average 0.69 and, ***as it happened to be***, for the X-Drivers, the PPMC was 0.53. Further, for these X-Variates 80% had PPMC-coefficients that were >0.

These associational-conditional selection screens for accruing the {Y🡐X}-Panels will greatly facilitate the de-construction of the Extrapolation-Impact of the four testing cells of Table 1.

3.1.3 Preview *En Bref* of the Cells of Table 1

Technically, Extrapolations are ***only in evidence*** when X_{PPV} is NOT IN the ordered-interval of the X-Values used to create the OLSR-Model. However, for completeness ***and because we have conditioned the Y-Variates to exhibit non-trivial Pearson Product Moment Correlation [PPMC] with the Standard Time-index***, we are very interested in all of the ***quasi***-Extrapolation-effects in:

Cell[2,2]: **[X]Ext & [Y]Ext** Case: Here Both the $X_{Proposed}$*-Projection-Value* [X_{PPV}] *and the* "*Next to be observed Y-value*"[Y_{Next}] are **NOT IN** the respective ordered X-Panel Set or the ordered Y-Panel Set used to create the OLSR Forecasting Equation Eq[1],

Cell[2,3]: **[X]Ext & [Y]Int** Case: Here the [X_{PPV}] is **NOT IN** in the ordered X-Panel Set ; while, the [Y_{Next}] **IS IN** the ordered Y-Panel Set used to create the OLSR Forecasting Equation Eq[1].

Cell[3,2]: **[X]Int & [Y]Ext** Case: Here the [Y_{Next}] is **NOT IN** in the ordered Y-Panel Set ; while, the [X_{PPV}] **IS IN** the ordered X-Panel Set used to create the OLSR Forecasting Equation Eq[1].

Further, as a vetting check, we expect that the 95%PI for Cells [3,3]: [X_{PPV}[Int] & $Y_{X_{PPV}}$ [Int]] will conform to the mathematical-expectation as noted above. Thus, we will use the results for Cell [3,3]: [The **[X]Int & [Y]Int** Case] as a vetting-test.

4. The Testing Datasets: Associational-Blocking of the Testing Framework

4.1 Testing Profiles

We selected Trading Market Firm Datasets as these have ***natural*** {Y🡐X}-profiles usually formed by associational-generating processes. The selection of these Datasets is detailed following: We:

I. randomly sampled 60 firms traded on the S&P$_{500}$ that are detailed on the *Bloomberg*™ Market Trading Platform [BBT],

II. selected as the Y-Panels for the OLSR-Model, the: BBT[*Stock Value*]. *Selection Screen*: These Y-Panel were only selected if their PPMC with [The Unit-Time Index] was Positive & was ≥ 0.35,

III. benefited from collegial-discussions regarding logical X-drivers of the Y[*Stock Value]* listed on: The BBT[*Income Statement* [GAAP-version] & The BBT[*Multiples Platform*]. This resulted in a total 18-possible X-Drivers, from which, we selected five *each* from these two X-Driver BBT-platforms. See Table 2. So as ***permit*** the opportunity to have variation in the X-Variable-profiles, we did not conditionally select the ten X-Drivers based upon their associational-profiles,

IV. created OLSR [Y🡐X]-forecasts using the first 13-Panel {Y&X}-Values for each firm; for the ***X-value*** projections, we used: (i) the exact value of the firm's 17th Data-Point [$X_{PPV[17]}$] & (ii) the exact value of the firm's 21st Data-Point$X_{PPV[21]}$, These, X-points were used create the 95%PIs & BMP[95%PIs], finally

V. selected as: the Y_{Next}, the ***Y-value matched*** with the selected $X_{PPV[]}$.

Table 2. The X-Drivers: From the BBT-Platform Income St [GAAP] & Multiples

Selected ***X-Driver Variates*** for ***Y-Stock Value***	BBT Platform
[1]AVERAGE_PRICE_EARNINGS_RATIO	Multiples
[2]AVERAGE_PRICE_TO_FREE_CASH_FLOW	Multiples
[3]PX_LAST	Multiples
[4]ENTERPRISE_VALUE	Multiples
[5]AVERAGE_PRICE_TO_SALES_RATIO	Multiples
[6]SALES_REV_TURN	Income St[GAAP]
[7]EARN_FOR_COMMON	Income St[GAAP]
[8]GROSS_MARGIN	Income St[GAAP]
[9]EBITDA	Income St[GAAP]
[10]PROF_MARGIN	Income St[GAAP]

For the OLSR Model only the first 13 Quarters were used, starting with the earlier of the 3rd or 4th Quarter 2014. We selected this as the starting time index as it was more than five years after the *Lehman Bros*™ Sub-Prime financial debacle[iii] that almost crashed the world's trading markets. We judged that this was a sufficient time-lag for the Markets to re-adjust after the 2008 Lehman-Event. See LinkedIn™ for a discussion of issues re: The Recovery[iv] from the Lehman-Debacle. We selected 13-quarters as this was a Panel of sufficient length for fitting an OLSR-Model. See Adya and Lusk (2016, p. 74).

4.2 Paring the 60 Selected Firms

The reasons for ***elimination*** of a Firm ***or*** of the [Y🡐X]-OLSR from the randomly accrued 60-firms were:

Note: If the Panel to be eliminated were to have been The Stock Value[Y-Variable] that ***Firm*** *would be eliminated; if the Panel were to have been one of the ten X-Variables only the OLSR would have been eliminated.* ***Elimination*** resulted for:

I. Instances where there were any missing Panel-values. [*We did not Regression- nor Near-Neighbor-fill these missing-values*],

II. [*Referencing Table 2*]: Cases where a selected X-variable or a selected Y-variable of a particular firm was not part of the firm's BBT[AIS-Ledger Set [See Table 2]]**,** or

III. Cases where the PPM-Correlations of: [*The Y-Panel used to fit the OLSR* (*w.*) The Unit-Time Index] was < 0.35. This was evidence of a relatively weak associational-generating process for the Y-variate.

After applying these Panel elimination screens, we arrived at 33 firms [See Appendix B] and overall 330 forecasts for each of the Two- $X_{PPV[]}$s. An illustration is in order.

4.3 Illustration of the Computations: AAPL as Downloaded

Consider the two variates *APPLE,* Inc. See Appendix A.

The Y-Variate is Apple Inc (AAPL US) – ***STOCK_VALUE*** [PPMC[SV][+**0.54**]], and

The X-Variate is: Apple Inc (AAPL US) – [BBT: Multiples] ***AVERAGE_PRICE_EARNINGS_RATIO*** [PPMC[APER[**–0.18**]].

The number in **[]s** is PPMC with the Time-Index.

For our testing profile, we will:

I. use the first 13-Panel Points to fit the [Y←X] OLSR-Forecasting equation Eq[1],

II. create two 95%PIs & two BMP[95%PIs] using the two randomly selected X-Variates:

$X_{PPV[17]}$ & $X_{PPV[21]}$,

III. label these selected $Y_{Next[]}$s as ***Ext***rapolations ***or Int***erpolations, as is the case. If the selected $Y_{Next[]}$ is ***IN*** the ordered-range of the Y-Panel: [1, - - -, 13], then the codex for the [$Y_{Next[]}$] is: ***Int***erpolation, otherwise, ***Ext***rapolation

IV. label these selected $X_{PPV[]}$s as ***Ext***rapolations ***or Int***erpolations as is the case. If the selected $X_{PPV[]}$ is ***IN*** the ordered-range of the X-Panel: [1, - - -, 13], then the codex for the { $X_{PPV[]}$] is: ***Int***erpolation, otherwise, ***Ext***rapolation

V. determine the orientation of the [$Y_{Next[]}$s] that corresponds *to the* [$X_{PPV[]}s$] *re:* The two respective 95%PIs. In this case, **If** these [$Y_{Next[]}$]s *are **IN** the respective* 95%PIs, **Then** the codex for the [$Y_{Next[]}$] is: ***IN***, otherwise, ***OUT***, finally

VI. record this result in *SAS*™[*JMP*™$_{\text{v.13}}$ Database] for AAPL.

After aggregating all the tests for the OLSR[95%PI]- & OLSR[BMP[95%PI]]-trials that we are testing, they will be profiled, analyzed, and discussed.

4.3.1 Detailed Computations: AAPL[95%PI]

Following are all the computations needed to create this profiling information for the *AAPL*-dataset. Using the {Y & X} Panels in Appendix A, we have the following:

The [Y←X]-Forecasting Equation is:

$Y_{X_{PPV}]}$ = [33.6743 + [-0.5205 × $X_{PPV[]}$]].

The ordered Range of the Y-Panel [The first 13 values] used to parameterize the OLSR is:

[MinY[19.174] : MaxY[31.688]].

The ordered Range of the X-Panel [The first 13 values] used to parameterize the OLSR is:

[MinX[10.6301] : MaxX[17.3003]].

The two [$Y_{Next[]}$] *are*: [$Y_{Next[17]}$] = 42.308, *and* [$Y_{Next[21]}$]] = 39.058

The two [$X_{PPV[]}$]s are: {$X_{PPV[17]}$] = 17.6685 and $X_{PPV[21]}$] = 16.4138}.

In this case, $Y_{Next[17]}$ is an ***Ext***rapolation, and $Y_{Next[21]}$ is an ***Ext***rapolation. *Rationale*,

$Y_{Next[17]}$ = 42.308 $\not\subset$ [MinY[19.174] : MaxY[31.688], and

$Y_{Next[21]}$ = 39.058 $\not\subset$ [MinY[19.174] : MaxY[31.688].

In this case, $X_{PPV[17]}$ is an ***Ext***rapolation, while $X_{PPV[21]}$ is an ***Int***erpolation. *Rationale*,

$X_{PPV[17]}$ = 17.6685 $\not\subset$ [MinX[10.6301] : MaxX[17.3003], while

$X_{PPV[21]}$ = 16.4138 $\subset$ [MinX[10.6301] : MaxX[17.3003].

Given the forecasting equation and the two [$X_{PPV[]}$]s, we have the following two **95%PIs**:

For $X_{PPV[17]}$ = 17.6685: The 95%PI is: [14.8108 : 34.1437], and

For $X_{PPV[21]}$ = 16.4138: The 95%PI is: [15.9279 : 34.3328]

Recall that the two [$Y_{Next[]}$] *are*: [$Y_{Next[17]}$] = 42.308, *and* [$Y_{Next[21]}$]] = 39.058.

In this case, both $Y_{Next[17]}$] = 42.308, and [$Y_{Next[21]}$]] = 39.058 are **OUT** of their 95%PI.

The recording of these Profiles is:

Table 1 for $Y_{Next[17]}$ the entry is in Cell(2,2): [**X]Ext & [Y]Ext**

Table 1 for $Y_{Next[21]}$ the entry is in Cell(3,2): [**X]Int & [Y]Ext**

Note: The PPMC[SV w. APER] is: [–0.26] This inverse relationship is *the likely* reason for the resulting Table 1 classifications.

Consider now the ***Width*** of the Median Benchmarked Precision of the 95%PIs

4.3.2 Detailed Computations*: AAPL*[Width of the BMP[95%PIs]

The second test-result is to record the ***Median*** Benchmarked Width of the 95%PIs]—i.e., The BMP[95%PI] associated with the particular test. In this case, we will present the inferential profiles of the Median for the BMP[95%PI] for the cases in Table 1. In this regard, we offer the details of the computations needed to profile the BMP[95%PI]. As an illustration, following are the computations for the *AAPL*-Panels for the inferential profiles of the Median for the BMP[95%PI] for Cell [2,2]: **[X]Ext & [Y]Ext &** Cell [3,2]: **[X]Int & [Y]Ext**:

Computations: The Y-Median of the *AAPL* dataset is: 27.008. There are two 95%PIs:

$X_{PPV[17]}$ = 17.6685: The 95%PI is: [14.8108: 34.1437], and

$X_{PPV[21]}$ = 16.4138: The 95%PI is: [15.9279 : 34.3328].

Thus, there are two BMP[95%PI] as follows:

BMP[95%PI[$X_{PPV[17]}$]]= 0.35791 [[[34.1437 – 14.8101]/2] / 27.008]

BMP[95%PI[$X_{PPV[21]}$]]= 0.34073 [[[34.3328 – 15.9279]/2] / 27.008]

Note: The ***Width*** of these 95%PIs and the ***Widths*** of the BMP[95%PIs] follow the same order relationship. Specifically, **If** the Width of 95%PI[$X_{PPV[17]}$ is > the Width of 95%PI[$X_{PPV[17]}$, **Then** the Width of BMP[95%PI[$X_{PPV[17]}$]] ***WILL BE*** > the Width of BMP[95%PI[$X_{PPV[21]}$]]. The reason for this is that: (i) both of the X_{PPV}-values are on the same side of the $\bar{x}$=26.2266 & (ii) the Y-Median is a positive Constant—which is the case for all the accrued Y-Panels.

4.4 Recoded Profile for AAPL

We are interested in three features for each evaluation event. [Shaded in Table 3] These are recorded in the JMP-DataBase as follows:

Table 3. Capsule of the JMP-Database used to Produce the Inferential Profiles

Firm Tested	X[Points]	Cells Table 1	BMP[95%PI]	Capture Rate [IN or OUT]
AAPL [SV & APER]	$\boldsymbol{X}_{PVV[17]}$	[X]Ext & [Y]Ext	0.35791	OUT
AAPL [SV & APER]	$\boldsymbol{X}_{PVV[21]}$	[X]Int & [Y]Ext	0.34073	OUT

All of the 660 OLSR Model data-points will be captured in this format and then analyzed.

5. Analysis of the Table 1: Suggested Profile of Extrapolations

5.1 Overview

Our experiential evidence over many years of consulting engagements and in an advisory capacity is clear: Almost exclusively, the Forecasting Division [*or Outsourced Forecasting Firm*] of organizations is a Staff-Support-Group in the Organization Hierarchy; implication—The Forecasting Division is not a Line Decision-Maker. Typically, they service the Planning Collective of the Organization: Usually composed of: Marketing-, Procurement- & Production- Line-decision-makers. This collective, *in almost any scenario imageable*, is interested ***in Forecasts of future activity both: Tactical*[*short-term*] *&* *Strategic*[*long-term*]**. In most cases, this requires the Forecasting Division to select $X_{proposed}$-values that are used to generate: [The Forecasts & The related [1-α]Prediction Intervals] of the ***future activity*** which are then forwarded, in a timely fashion, to the Planning Collective as input to their Planning deliberations. In our experience, often the Forecasting Group makes a *PowerPoint*™ Presentation to the Planning Committee and thus, the Forecasting Group is an information-source ***invited*** to participate in the deliberations of the Planning Committee. Predominantly, the $X_{proposed}$-values that are used to create these Profiles of future activity used to inform the deliberations of the Planning Committee are usually ***Extrapolations***; most often [Cell[2,2]-[X]Ext & [Y]Ext] Extrapolations.

We have discussed with our contact-forecasting colleagues, the following question:

What is the Nature of the Jeopardy of using Extrapolations in creating Forecasting-Profiles of the Future Actively that are communicated to the Planning Committee?

Often, we received the following feedback:

Extrapolations-issues are not practical issues for the Real-World of organizational planning; of course our forecasting-projects are Extrapolations—What else could they be? However, to be sure, these are certainly of interest in theoretical investigations.

5.2 Ok! Reality Confession

Before we undertook this research project, we were unaware of the ***actual magnitude*** of the impact of Extrapolations re: The Capture Rate of the 95%PIs or the Width of the BMP[95%PI[X_{PPV}]]. This is, of course, the nagging question that motivated this research-report. As a very interesting addendum: We conducted a *ProQuest*™[Global]*ABI/INFORM*™ [AND/OR] search for: "[*Extrapolations*] : [*Prediction Interval*] : [*Precision*]" and found ***no Peer-Reviewed citations***. [14March2024]. This suggests that the Profiling of Tables[1: 4: 5] will be an addition to the effect of Extrapolations in the forecasting domain.

With this overview, we now offer, for the first time in a Peer-Reviewed outlet, the four-Profiles of Table 1. As a preamble to discussing Tables [4 & 5], we offer a set of three ***Vetting Tests***. This is a necessary testing phase that is needed in all inferential studies to offer an indication of the generalizability of the inferential results to be presented.

5.3 Vetting Screens: Addressing the Generalizability and Utility of our Profiling of Table 1

Vetting screens are ***simple***, and to be sure ***intuitive***, inferential-analyses that test common-sense relationships that should be expected; if they are not in evidence, this would call into question the validity of the inferential results of the study. For this exploration of the impact of the Extrapolation as profiled in Table 4, we offer three vetting tests.

5.3.1 Vetting I

For Table 1, Cell[3,3]:**[X]Int & [Y]Int** should, in theory, have a ***Capture Rate*** for each of: $\boldsymbol{X_{PPV[17]}}$ ***and*** $\boldsymbol{X_{PPV[21]}}$ re: the respective: $\boldsymbol{Y_{Next[17]}}$ & $\boldsymbol{Y_{Next[21]}}$, ***of 95%.***

Results Vetting I

Cell[3,3]: **X[Int[& [Y]Int[**X17**]**→95%CI [94.2% : 100%] Mean [98.04%]

Cell[3,3]: **X[Int[& [Y]Int[**X21**]** →95%CI [90.6% : 100%] Mean [96.00%]

As expected, given the theory underlying the OLSR-Model & the 95%PIs, it is the case that the 95% Confidence Interval of the ***Mean of the Capture Rates*** for [**X]Int[& [Y]Int]** for both the $X_{PPV[17]}$ *and* $X_{PPV[21]}$ contain the expectation of 95%. ***Summary*** *The Capture Rate for Panels classified* **X[Int[& [Y]Int** *follows the theoretic expectation.* ***Thus, Vetting I is founded.***

5.3.2 Vetting II

Given T&D's comment about the effect of Extrapolations on the ***Width*** of the PIs, we proffer that the

BMP[95%PI[X_{PPV}]] of: Cell[2,2]: **[X]Ext & [Y]Ext** should be **>[Wider/less precise]** than that of Cell[3,3]: [X]**Int & [Y]Int**}. ***Rationale*** ONLY Cell[3,3] has $X_{PPV[17]}$- *and* $X_{PPV[21]}$-values and their related $Y_{X_{PPV[17]}}$ $Y_{X_{PPV[21]}}$ that are ***IN*** the Panel-set used to create the 95%PI. Thus, Cell[2,2]: **[X]Ext & [Y]Ext** will have ***more*** BMP[95%PI[X_{PPV}]] -values further away from $\bar{x}$ than is the case for Cell[3 × 3].

Results Vetting II

Medians Analysis *In this case, for the* ***two*** *Median-difference-tests of the* BMP[95%PI[X_{PPV}]]-values *for the two* X_{PPV}s, *we used the Wilcoxon / Kruskal-Wallis Tests (Rank Sums) test p-values. The two respective p-values are: <0.0001* ***These profiles are Bolded following****:*

IntInt:Median[$X_{PPV[17]}$***] = 24.3%*** ***IntInt:Median[***$X_{PPV[21]}$***]= 31.8%***

ExtExt:Median[$X_{PPV[17]}$***] = 40.8%*** ***ExtExt:Median[***$X_{PPV[21]}$***]]= 40.4%***

Means Analyses *In this case, for the* ***two*** *Mean-difference-tests of* BMP[95%PI[X_{PPV}]]-values *using Welch (1951)ANOVA test, the two respective p-values are: <0.0001. These are Italicized.*

IntInt:Mean[$X_{PPV[17]}$*] = 24.7%* *IntInt:Mean[*$X_{PPV[21]}$*] = 30.4%*

ExtExt:Mean[$X_{PPV[17]}$*] =51.3%* *ExtExt:Mean[*$X_{PPV[21]}$*]= 50.4%*

Summary *Note: IntInt is Cell(3,3) and ExtExt is Cell(2,2). For the* BMP[95%PI[X_{PPV}]]-values *for both the Medians as well as the Means there is uniform and strong inferential evidence that the ExtExt-screen has wider* BMP[95%PI[X_{PPV}]]-values *compared to that of the IntInt-screen controlling for the selected points* $X_{PPV[17]}$ *&* $X_{PPV[21]}$. ***Thus, Vetting II is founded.***

5.3.3 Vetting III

We will inferentially evaluate the BMP[95%PI[X_{PPV}]]-values as a ***reasonability*** screen for the accrual of 33 firms and 660 OLSR Forecasts. In this case, our experimental evidence in many other forecasting-studies, is that the BMP[95%PI[X_{PPV}]]-values were in the Range [25% : 60%]. Thus, we examined the ***Full datasets*** for the Means of the BMP[95%PI[X_{PPV}]]-values *for the selected points* $X_{PPV[17]}$ *&* $X_{PPV[21]}$.

Results Vetting III

For the Full-Data sets for each of the selected points $X_{PPV[17]}$ *&* $X_{PPV[21]}$ *for the samples n=330 each, we found the following* ***95%CIs*** *for the* ***Means*** *of the* BMP[95%PI[X_{PPV}]]-values:

$X_{PPV[17]}$ 95%CI→[35.6% : 46.2%]; note that this 95%CI is IN the expectation-Range [25% ; 60%], and

$X_{PPV[21]}$ 95%CI→[36.2% : 46.8%]; note that this 95%CI is IN the expectation-Range [25% ; 60%].

Summary This is strong evidence that these 33 Firms and the 330 forecasts for each of the X-Points selected are not likely to be inferentially-[Non-Ergodic] with respect to the experiential-Range [25% : 60%] offered. ***Thus, Vetting III is founded.***

5.4 Overall Summary of the Vetting-Profiles

We offer these three vetting Profile-tests as reasonable benchmarks for comparing our random sample from the S&P$_{500}$ to a population of Market Trading Firms. For each of these vetting inferential-based tests, we judge the Vetting-Tests to be founded. ***Thus, these vetting-profiles offer reasonable evidence that our S&P$_{500}$ accrual is not an unreasonable draw from a Market Trading Population.***

6. Testing of the Extrapolation Impact

6.1 Overview

Given the acceptance of the vetting results that there is evidence of the generalizability of the results, of course—assuming the logic of the following inference tests and their interpretation, we offer the details of the profiling of Table 4 that addresses the pivotal question:

What is the Intel that is offered by ***The 95% Prediction Interval***?

6.2 The Inference Profiles of the Capture Rates

Following in Table 4 are the Capture Rates Results for the four Cells in Table 1. Recall, these Capture Rates Results are for the X-Variate Points: $[X_{17}]$ & $[X_{21}]$ and ***the related*** 95%PI where the “assumed next observed” Y-Variate Points: $[Y_{17}]$ & $[Y_{21}]$ are the actual firm values paired with the X-Variate Points: $[X_{17}]$ & $[X_{21}]$. These results are presented for the four Cells: {[X]**Ext & [Y]Ext** : [X]**Ext & [Y]Int** : [X]**Int & [Y]Ext** : [X]**Int & [Y]Int**} of Table 1.

Table 4. Profile Elaboration of Table1

Percentages **IN** the 95%PI	**Capture Rates Point 17**		**Capture Rates Point 21**	
	$[Y_{Next}]$-Value[Ext]	$[Y_{Next}]$-Value[Int]	$[Y_{Next}]$-Value[Ext]	$[Y_{Next}]$-Value[Int]
$X_{PPV[]}$ **Ext**	**[X]Ext & [Y]Ext**	**[X]Ext & [Y]Int**	**[X]Ext &[Y]Ext**	**[X]Ext & [Y]Int**
Capture Rate	44.81% [69/154]	100.00% [19/19]	51.53% [84/163]	90.00% [18/20]
95% CI	[37.0% : 52.7%]	N/A	[43.9% : 59.2%]	[76.9% : 100%]
p-value [v.95%]	<0.0001	N/A	<0.0001	0.01
$X_{PPV[]}$ **Int**	**[X]Int & [Y]Ext**	**[X]Int & [Y]Int**	**[X]Int & [Y]Ext**	**[X]Int & [Y]Int**
Capture Rate	24.53% [26/106]	98.04% [50/51]	22.68% [22/97]	96.00% [48/50]
95% CI	[16.3% : 32.7%]	[94.2% : 100%]	[14.3% : 31.0%]	[90.60% : 100%]
p-value [v.95%]	<0.0001	0.09	<0.0001	0.57

The deconstruction of the information presented in Table 4 is very interesting re: The Capture Rates for the X-Variate Points: $[X_{PPV[17]}]$ & $[X_{PPV[21]}]$.

6.2.1 Individual Profiles: Capture Rates 95%PI: Table 4

Caveat Interestingly, the **{X]Ext & [Y]Int}**-Tests for Points: $[X_{PPV[17]}]$ & $[X_{PPV[21]}$ [Shaded] did not result in sufficient activity such that they have a meaningful inferential interpretation. The number of trials for the **[X]Ext & [Y]Int**-screens **for both** Points: $[X_{PPV[17]}]$ & $[X_{PPV[21]}]$ had only about 6% [20/330] each of the S&P$_{500}$ forecasting-trials. However, these Cells are interesting from an experiential-judgment perspective. Specifically, the **[X]Ext & [Y]Int**-screen suggests that there are very few instances where there is a $[Y_{Next}]$ that is associated with *an* $[X_{PPV}]$ that is an **Ext** where the associated $[Y_{Next}]$ is an **Int**erpolation. This is very likely due to the associational-generating processes of these Market Trading Panels—this circumstance is sometimes referred to as: positive-associational-drift. ***Simply, where there is the possibility of positive-associational-[Y←X] drift in the sampled population, such as is the case for our S&P$_{500}$ sample, the*** $[Y_{Next}]$***-value often follows the*** X_{PPV}***-value and thus an [X]Ext will likely have a*** $[Y_{Next}]$***-“partner” that is likely to also be a*** $[Y_{Next}]$***-Ext.*** Thus, it would be ***rare*** for there to be a **Y[Int]** associated with an **X]Ext**] as is the case for Table 4**.** As a final note, for completeness, the average percentage of the activity for the **[X]Ext & [Y]Int-screens for both** Points: $[X_{PPV[17]}]$ & $[X_{PPV[21]}]$ is inferentially lower [p-value <0.0001] than the average percent of the activity for the other two-Cells in the $X_{PPV[]}$ **Ext row.**

6.2.2 Inferential Analysis

For the **[X]Ext & [Y]Ext- and the [X]Int & [Y]Int**-screens for each of the Points: $[X_{PPV[17]}]$ & $[X_{PPV[21]}]$, the 95%PI Capture Rates follow the logic of strong positive-associational-drivers where: **IF** the $X_{PPV[]}$ is an **Ext, THEN** it is not unexpected that the associated-$[Y_{Next}]$ also is likely to be an **Ext** and, by definition, this $[Y_{Next}]$ is thus likely to be outside the 95%PI as is indicated by the low-Capture Rate in Table 4. Of course, the expectation is *visa-versa* for the case of the **[X]Int & [Y]Int** Profile. ***Rationale*** Where there is pervasive evidence of **Ext**rapolations, the 95%PI Capture Rates are inferentially far below the 95% Expectation. Note: The p-values of the test against of the 95% expectation are <0.0001. For example, for [$X_{PPV[21]}$] for the **[X]Ext & [Y]Ext**-domain for which we have the following inferential computation:

z_{Cal}= 24.8 = ABS[84/163 – 95%] / [(95% × 5%)/163]^0.5

that gives a p-value <0.0001. This clearly indicates the rejection of the FPE[Null] of equality.

At the other polar-position, as presented for Vetting Test I, for **the [X]Int & [Y]Int**-screens, the 95%Confidence Intervals of the Mean of the Capture Rates contain 95% indicating that for both Points: [$X_{PPV[17]}$] & [$X_{PPV[21]}$], the Capture Rates for the [Y_{Next}] conform to the Theoretical Expectation of the 95%PIs.

Finally, the **[X]Int & [Y]Ext**-screens, are sufficiently populated to warrant inferential-testing and profiling for both of the [$X_{PPV[17]}$]- & [$X_{PPV[21]}$]-Variates. These profiles suggest that there are a large number of instances: [106 & 97 respectively] where the [Y_{Next}] associated with the $X_{PPV[]}$ [**Int**] is NOT IN the Y-Panels used to create the 95%PI. This is the reverse of the **[X]Ext[& [Y]Int** profile. ***Rationale*** This occurs due to the fact that $X_{PPV[]}$ is **IN** the Data Range of the X-Variate. Thus, there are two effects: (i) The Precision of the 95%PI is likely relatively more-narrow than if the $X_{PPV[]}$ were to have been an **Ex**trapolation—more likely to exclude any test value, and (ii) as the [Y_{Next}], in most cases, is being driven by a stronger-directional associational-process relative to that driving the $X_{PPV[]}$-variates, this acts as a longitudinal-"push" for the trajectory of the values in the Y-Panel. Thus, this latter-condition acts to create the environment where usually the [Y_{Next}]-points are at some ***distance*** from the temporal-location of the $\bar{x}$ and thus will fail to be IN the Y-Panels used to create the 95%PI. This effect and the former effect for the more -narrow-width of the 95%PI will result in the very low Capture Rate for [Y_{Next}] even though the X-Proposed is an [**Int**]. This is an interesting and logical, and on our part, an unanticipated result.

6.3 The Extrapolation-Effect on the BMP[95%PI[X_{PPV}]]-values

We have evidence from Table 4 that overall Extrapolations result in excessive Failure to Capture relative to the expectation of the 95%PI[X_{PPV}]. In addition, anomalously they often widen the BMP[95%PI[X_{PPV}]] so that they are ***less precise and so less informative***. To profile this second Extrapolation-issue, we have indicated the Table 4 Cell partitions for the various BMP[95%PI[X_{PPV}]]-values in Table 5.

Table 5. Precision Profiles and Testing of the Median BMP[95%PIs]

Panel Point [Y & X] 17	Median BMP[95%PIs]	Mean BMP[95%PIs]	Median p-values < 0.01	Mean p-values < 0.01
ExtExt n=154	37.8%	52.8%	←v.IntInt	←v.IntInt
ExtInt n=19	35.0%	39.5%	N/A	N/A
IntExt n=106	28.1%	31.7%	←v.ExtExt	←v.ExtExt
IntInt n=51 **Sum 330**	24.1%	24.6%	←v.ExtExt	None
Panel Point [Y & X] 21	Median BMP[95%PIs]	Mean BMP[95%PIs]	Median p-values < 0.01	Mean p-values < 0.01
ExtExt n=163	36.8%	46.8%	←v.IntExt	←v.IntInt
ExtInt n=20	37.9%	42.4%	N/A	N/A
IntExt n=97	27.4%	38.2%	←v.ExtInt	None
IntInt n=50 **Sum 330**	31.7%	30.0%	←v.ExtExt	None

6.3.1 Table 5 Codex

We computed for the two-points tested [$X_{PPV[17\,\&\,21]}$] the Median and the Mean for the BMP[95%PIs] for each of the four Table 4 Partitions. These are noted in Col[1] along with the sample-sizes—also used for the Capture Rate Profiles of Table 4. Recall: ***Wider Confidence Intervals*** offer less-precise thus, are less-useful intel; this indicates that being in a very wide 95%PI is likely to NOT be desirable! For the two Extrapolation components shaded there were insufficient instances to form reliable inferential intel; thus, they are excluded from our deconstructive analysis. In columns [4] & [5], we have coded the inferential ***pair-wise p-values***. For example, for BMP[95%PI[$X_{PPV[17\,]}$]] for Cell[]2,2] **[X]Ext & [Y]Ext** of Table 1, there were 154 instances; the Median of which was: 37.8%. In Col[4], we note that the pairwise-test using the Wilcoxon Method[v] for the following comparison:

{[X]Ext & [Y]Ext [37.8%]} against {**[X]Int & [Y]Int** [24.1%]}

had a non-directional p-value <0.01.

6.3.2 Suggested Screening-Information from Table 5

In this case, it is inferentially clear for the Median & the Mean for both $X_{PPV[17\,]}$ & $X_{PPV[21]}$ that:

[X]Ext & [Y]Ext IS wider [less precise] than is **[X]Int & [Y]Int**

Further, ONLY **[X]Int & [Y]Int** and [**X]Int & [Y]Ext** are computed correctly—i.e., in accordance with the theoretical requirements of the OLSR Model—re: forming the 95%PI[$X_{PPV[17\,\&\,21]}$] and the BMP[95%PI[$X_{PPV[17\,\&\,21]}$]]. The reason for this is that $X_{PPV[17]}$ & $X_{PPV[\,21]}$ in conjunction with the OLSR-parameters fix the nature of: The 95%PI & The BMP. Thus, by definition, See Tamhane & Dunlop (2000, p.263), any $X_{PPV[]}$ that is an ***Ext***rapolation will not result in a correctly computed 95%PI[$X_{PPV[]}$] & BMP[95%PI[$X_{PPV[]}$]]. ***Note*** The Capture Rates may be very different for [X]Int **& [Y]Int** and [X]Int **& [Y]Ext;** this difference is due to the fact that the **[Y]Int** comes from the same sample-set that was used to create the OLSR & The 95%PI & The BMP. Whereas, **[Y]Ext** comes from ***outside*** the Y-Panel used to fit the OLSR & The 95%PI & The BMP. *Voilà*, the difference in the **Capture Rates.**

7. "Take-Away" Re: The Capture Rate & The Benchmarked Precision

7.1 The Capture Rate Profiles: Associational-Context

When the Forecasting-Profile is: [$X_{PPV[]}$]Int & [Y_{Next}]Int, the Theoretical Expectation for the 95%PI is the likely State of Nature; to wit: 95% of the time the [Y_{Next}] *will likely be somewhere in the* 95%PI. Otherwise, not so much the case. Specifically, IF the Forecasting Protocol is: [$X_{PPV[]}$]**Ext** ∪ {[[Y_{Next}]Ext ***or*** [[Y_{Next}]]Int}, THEN this is, by definition, **a Computational Error** with respect to the Math/Stat assumption-set of the OLSR Model. This will, as presented in Tables [4 & 5], compromise the 95%PI and in ***an associational-context*** will result in a Capture Rate inferentially lower than 95%.

7.2 Benchmarked Precision BMP[95%PI[X_{PPV}]] : The Associational-*Context*

By the T&D-definition, the ONLY "computationally correct precision" is created by any $X_{PPV[]}$ that is an ***Int***erpolation. This ***will*** result in a correctly computed BMP[95%PI[$X_{PPV[]}$]—i.e., the test is Simple**: If the $X_{PPV[]}$ is an *Int*erpolation this will usually provide the *best or* [*MLE*] *precision*.**

8. Summary & Outlook

8.1 Summary

Navigating the decision-making world of Forecasters is inherently fraught with difficulty. However, when Forecasters use ***Extrapolations***, in creating forecasting-intel that is supposed to be relevant and useful for informing the decision-making process, they are willing participants in exacerbating the inherent difficulty. Following, we offer a simple Decision Alert-Grid to assist Forecasters who endeavor to: "***Do the Right Thing***" when dealing with {Y🡐X}-Panels that have Associational-Drivers not unlike those that were in evidence for the S&P$_{500}$-Panels.

Table 6. Final Summary of the Tamhane & Dunlop Comment: **Extrapolations are Risky Business**

Forecasting Cell Profile ***Table 1***	***Capture Rate Profile*** 95%PI[$X_{PPV[17\ \&\ 21]}$]	***Median Benchmarked Precision*** BMP[95%PI[$X_{PPV[17\ \&\ 21]}$]
Cell (2,2) **[X]Ext & [Y]Ext**	*Issue Alert*	*Issue Alert*
Profile Take Away	*Incorrect 95%PI* *Low Capture Rate [≈50%]*	*Very wide BMP ≈twice the width of the BMP of the {Int&Int}-version*
Overall Indication	***Avoid $X_{PPV[]}$ Extrapolations in any Forecasting OLSR-Context***	
Cell (3,2) **[X]Ext & [Y]Int**	*Issue Alert*	*Issue Alert*
Profile Take Away	*Incorrect 95%PI* *The* associational-*context results in a dearth of activity*	*Very wide BMP ≈ 1.5 x the BPM-width of the {Int&Int}-version*
Overall Indication	***Incorrect PIs & BMPs Low activity incidence due to Associational-Drift***	
Cell (2,3) **[X]Int & [Y]Ext**	*Issue Alert*	*Issue Alert*
Profile Take Away	***Correct 95%PI*** *The* associational-*context results in a* **very low** *Capture Rate [≈25%]*	*Not significantly different than the BMP of the {Int&Int} Re: Means or Medians for p<0.1*
Overall Indication	***Correct PIs & BMPs Low Capture Rate due to A***ssociational-***Drift***	
Cell (3,3) **[X]Int & [Y]Int**	*Issue Alert*	*Issue Alert*
Profile Take Away	*Correct 95%PI* *Even for the* associational-*context the Capture Rate [≈95%]*	*Correct BMP* *For the p-value tests Int&Int was* **usually** *not outperformed Re: p-values.*
Overall Indication	***Gold Standard Profile*** *for: The* 95%PI & *The* BMP[95%PI]	

Discussion The Overall Summary Table 6 confirms the wisdom of the Tamhane & Dunlop (2000, p. 363) [T&D] advice re: forecasting:

> *"- - -extrapolation beyond the range of the data is a* **risk business** *should be avoided.*

Indeed, this is the case. However, there is a nagging behavioral glitch. Given the ***critical*** importance given to forecasting the future activity of the firm by the Senior Planners of the Firm, it seems that ***Extrapolations*** will continue as the "*Mode d'Emploi*" for the forecasters. As a forecasting colleague for a MNC quipped:

Yes, as most of us know, Extrapolations are the Bane of the Process of Informing the users of the forecasts of the level of confidence that they can "count on" in the near future. However, Extrapolations are unavoidable under the ***condition that I would like to be relatively sure of remaining as the Director of my firm's Forecasting Division****.*

8.2 Outlook

The associational-context that we have selected is typical for Market Trading Firms. Many such firms are driven by processes resulting in Panel-Points that are longitudinally associated. Thus, we tweaked our study by ***requiring*** that the Response Variable [$Y_{PPV[]}$] must have a PPMC-coefficient w. The Time-Index that is ≥0.35—a non-trivial association profile for the Y-Panel-Points. Additionally, we did not impose any PPMC-conditions on the X-Driver Variables [$X_{PPV[]}$]. In fact, about 20% of the Drivers had PPMC < 0—i.e., an inverse trajectory relative to that of the [$Y_{PPV[]}$]. The reason for this decision was to control or focus the experimental-context for profiling and discussing the Extrapolation-effect on: The 95%PI[] & The BMP[95%PI[]. In this regard, these PPMC-controls certainly seemed to highlight the Extrapolation-effects for this very typical Market Trading context as summarized in Table 7.

8.2.1 Extensions: Expansions

As a practical, but challenging extension, it would productive and informative, to examine Extrapolation-effects in the following enhanced-testing context as suggested by the following Matrix[4×4]: {Vector [$X_{PPV[]}$] $\otimes$ Vector [$Y_{PPV[]}$]:

Table 7. Proposed Extrapolations v. Interpolations testing Grid for the PPMC: noted as $\hat{\rho}$

Vector [$X_{PPV[]}$]	Vector [$Y_{PPV[]}$]
$X_{PPV[]}$]	$Y_{PPV[]}$]
$-1 \le \hat{\rho} < -0.35$	$-1 \le \hat{\rho} < -0.35$
$X_{PPV[]}$]	$Y_{PPV[]}$]
$-0.35 < \hat{\rho} \le 0$	$-0.35 < \hat{\rho} \le 0$
$X_{PPV[]}$]	$Y_{PPV[]}$]
$0 < \hat{\rho} \le 0.35$	$0 < \hat{\rho} \le 0.35$
$X_{PPV[]}$]	$Y_{PPV[]}$]
$0.35 < \hat{\rho} \le 1$	$0.35 < \hat{\rho} \le 1$

The $\otimes$-Function is the expansive-multiplication of the two-vectors in Table 7 that creates the 16 Cells of a Matrix[4×4]. If this Matrix[4×4] were profiled in the same manner as was Table 1, this would be a very welcomed ”***GPS***” for Forecasters.

8.2.2 Extensions

In addition, there should be a way to use ***Extrapolations*** in the forecasting context and then to apply Judgmental Adjustments or Informational Contextual Hedges to bring the Confidence Intervals ***back into line with their theoretical expectations***. This is a critical project; in this regard we invite collaborative contacts.

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References

Adya, M., & Lusk, E. (2016). Time series complexity: The development and validation of a Rule-Based Complexity scoring technique. *Decision Support Systems, 83,* 70-82. http://dx.doi.org/10.1016/j.dss.2015.12.009

Box, G. E. P., & Jenkins, G. (1970). *Time Series Analysis: Forecasting and Control*. Holden-Day.

Collopy, F., & Armstrong, J. S. (1992). Rule-based forecasting: Development and validation of an expert systems approach to combining time series extrapolations. *Management Science*, *38*, 1394-1414.

Makridakis, S., Andersen, A., Carbone, R., Fildes, R., Hibon, M., Lewandowski, R., Newton, J., E. Parzen, E., & Winkler, R. (1982). The accuracy of extrapolation (Time Series) methods: Results of a forecasting competition. *Journal of Forecasting*, *1*, 111-153. http://dx.doi.org/10.1002/for.3980010202

Tamhane, A., & Dunlop, D. (2000). *Statistics and data analysis*. Prentice-Hall.

Welch, B. (1951). On the comparisons of several mean values: An alternative approach. *Biometrica*, *38*, 330-336.

Appendix A

Table A1. Apple Inc (*AAPL US*) - Stock Value

20.003	19.174	22.995	25.188	28.498	30.813	31.688	28.678	27.008
26.418	23.35	28.178	28.955	35.915	36.005	38.53	42.308	41.945
46.278	56.435	39.058	47.488	49.48	54.705	72.45	61.935	88.408
112.28	131.97	121.21	133.11	146.92	176.28	174.72	141.66	150.43

Table A2. $X_{Proposed}[X_j]$-Values for: Apple Inc (AAPL US) - MultiplesAVERAGE_PRICE_EARNINGS_RATIO

14.205	15.8548	16.8732	16.2285	15.8191	13.6046	12.4375	10.6301	11.2117
12.281	13.798	15.7579	17.3003	17.6426	18.3375	17.8835	17.6685	19.084
16.4314	13.9595	16.4138	17.7836	21.5246	23.318	23.9945	32.8867	36.4764
34.9316	28.9599	28.7979	27.6679	27.9047	24.9881	25.8411	23.4023	25.0511

Appendix B

Table B1. BBT[BICS-Codes] For the Final $S\&P_{500}$ Firms

AAPL	AIG	AMZN	APH	AVGO	BBWI	BRK/A	EMR	FTNT	GD	GM
GOOGL	HAS	HD	JCI	JNJ	LLY	MAR	MCHP	META	MPC	MRK
MSFT	MSI	NVDA	NWSA	PARA	PG	PH	ROP	SEE	UNH	XOM

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[i] We also tested the tendency for organizations in the Production & Sales Sector to have reported production activity that is driven by processes that over time are PPMC with the Time-Index. For a random sample of 30 such firms that were part of the M-Competition Panel-set [181 Annual Series[Makridakis *et al.* (1982)]] the Mean of these PPMCs was 66.6% [4 Series had PPMCs < 0.]. This fits well and thus, is a vetting indication, that longitudinal-association of Production & Revenue & Stock Price seem to have the same generation-function characteristics.

[ii] These longitudinal-associations are usually produced by Multiplicative: [Collopy & Armstrong (1992] or ARIMA[Box & Jenkins(1970]: generating processes. We are only interested in the related measure of the PPMC of the Panel under examination with the related Time-Index for classification purposes. Whether the longitudinal-associations effect was created by a Multiplicative- or an ARIMA-Process is, for our context, a moot point.

[iii] https://www.investopedia.com/articles/economics/09/lehman-brothers-collapse.asp

[iv]

https://www.linkedin.com/advice/0/how-did-lehman-brothers-collapse-impact-global-economy-bsyac#what-experts-are-saying

[v] For the Median Pairwise Non-Parametric tests, we used The Wilcoxon Method for the Wilcoxon / Kruskal-Wallis Tests (Rank Sums). For the Mean Pairwise-tests, we used the Ordered Differences Report from the Comparisons for all pairs using Tukey-Kramer HSD-measure. These platforms and all the details are found in *SAS*™[*JMP*™ $_{[v.13]}$].